TURBULENT PARTICLE TRANSPORT
IN TOKAMAK PLASMAS WITH IMPURITIES

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ABSTRACT

The turbulence driven by a ion temperature gradient, a mass shear-flow parallel to the magnetic field, and a impurity ion density gradient in plasmas with multiple ion species is studied in a sheared slab magnetic configuration. The turbulence drive from the temperature gradient and parallel shear-flow of the majority ion component is shown to be enhanced by the shear-flow and negative density gradient of the impurity ions. The particle diffusion induced by the turbulence is obtained with quasilinear fluid theory. Optimal transport parameters for an inward ‘pinch’ of the majority ions and the outward flow of the impurity ions are determined. The corresponding effective diffusion coefficients are computed. Correlations with tokamak experimental observations such as an isotope scaling of plasma confinement time are discussed.

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1. Introduction

For many years in the magnetic confinement fusion research, the correlation between turbulence driven by microinstabilities and particle, energy-momentum transport of plasmas has been a central issue in experimental and theoretical discussions. A thorough understanding of the physical mechanism of plasma turbulence and the associated transport process is of central importance to the control of anomalous particle and energy loss, as well as to the optimization of fusion reactor design. Due to the low to high confinement (L-H) mode transition, the tokamak performance has been greatly improved. This improvement is theoretically considered to be associated with the presence of an $E \times B$-velocity shear $dv_r/dr$, the influence of which has been studied extensively both in experiments and in theories. In addition to the $E \times B$ shear, however, results from the experiments on the JT-60U also show that plasma confinement is significantly improved in the region where a steep shear layer of parallel velocity exists. Theoretical explanation for such important experimental observations is rare. Also most of the studies about $dv_\parallel/dr$ are limited to the main ions (hydrogen and its isotope), overlooking the influence of impurity ion flows. Meanwhile, however, almost all of the velocities measured in experiments are those of impurities because of diagnostic limitations. This leads to an assumption in theoretical analysis that the parallel rotation velocities of the main ions are the same as those of impurity ions. Considering this, a special measurement for velocity profiles of the primary discharge gas ($\text{He}^{2+}$) and the impurities ($\text{C}^{6+}$ and $\text{B}^{5+}$) has been taken on the DIII-D tokamak, and a significant difference between the profiles has been found. Therefore, in studies of the turbulence induced by $dv_\parallel/dr$ and corresponding transport phenomena, it is necessary to take into account the impurity effects, and to give an explicit account in theory to the realistic problem as to whether the effects of $dv_\parallel/dr$ from the main ions and that from the impurities are the same or not.

On the other hand, it is known that the radial profile of impurity density in the boundary region has a strong influence on the particle and energy-momentum transport of plasmas. Results from the experiment also show a great difference between the transport of the impurities and the main ions. It is plausible to think that the parallel velocity shear and the
impurity profile are two important driving mechanisms in the development of edge turbulence. The effects of the impurity density gradient on the ion temperature gradient (ITG) driven mode\textsuperscript{1} as well as the ITG mode characteristics in the presence of a parallel velocity shear have been studied theoretically\textsuperscript{5–7}. However, only two among the three driving mechanisms, i.e. ITG, impurity density gradient and parallel velocity shear are considered simultaneously in these studies. It is necessary to take these three driving forces into account simultaneously in the study of the instabilities and related transport.

As for the radial loss induced by turbulence in tokamak plasmas, ion momentum and energy transport have been investigated extensively\textsuperscript{7,8} while the issue of turbulent particle transport is scarcely touched in theoretical studies. In addition, there exist discrepancies between the experimental observations and the predictions from drift wave turbulence theories in which density or temperature gradient is considered as the driving mechanism only\textsuperscript{9}. Furthermore, the experimentally observed phenomena on particle transport such as the particle 'pinch' effect of the main ions\textsuperscript{10}, the direction of the impurity ion flux\textsuperscript{11} and its dependence on the plasma parameters need theoretical explanation and physical understanding.

In order to study the problems mentioned above, the two-fluid plasma equation is adopted in a slab geometry with magnetic shear in this article. The development of turbulence in the presence of three sorts of driving mechanisms: impurity density gradient, parallel velocity shear and ion temperature gradient is studied. The emphasis is placed on the correlation between impurity effects and parallel velocity shear, and on the radial particle transport driven by turbulence. The studies show that the direction and amplitude of the ion flux are mainly governed by ion density and temperature profiles, velocity shears as well as the main ion species. These analytical results are consistent with the experimental observations on transport scaling\textsuperscript{9} and the direction of ion flux.

The advantage of the fluid model here is that the dispersion equation for the eigenvalue is an algebraic equation and that analytic expressions for the eigenfunction and fluxes can be obtained. These make the results qualitatively reliable and provide guidelines for kinetic studies. Nevertheless, the fluid description of these modes tends to give growth rates larger
than the more precise kinetic growth rates. Thus, kinetic effects may be important in impurity transport studies and will be considered in a separate work.

The remainder of this article is organized as follows: In Sec. 2, the dispersion equation in the presence of density and temperature gradients, and parallel velocity shears of both the majority and impurity ions is derived and discussed in detail in a slab configuration with magnetic shear. A comparison of the effect from $dv_{\parallel}/dr$ and $dv_{\perp}/dr$ is given. Also emphasis is placed on the study of the comprehensive influence of impurity and parallel velocity shear on the growth rate of the instability. In Sec. 3, the expressions for the radial ion flux and corresponding transport coefficient are given, and the parametric dependence of the flux on plasma parameters is discussed in detail. In Sec. 4, a brief discussion and a summary of the results obtained in this work are given.

2. Physical model and stability analysis

We adopt a two-fluid theory in a slab magnetic configuration, $B = B_0[z + (x/L_s)\hat{y}]$, where $L_s$ is the scale length of magnetic shear. The $x$, $y$, and $z$ directions in the sheared slab geometry are defined as the radial, poloidal, and toroidal directions in the tokamak configuration. $\hat{y}$ and $\hat{z}$ are the unit vectors in $y$ and $z$ directions. We assume a background plasma with all inhomogeneities only in the radial direction, where perturbations have the form $f(x)\exp(-i\omega t + ik_y y + ik_z z)$. The parallel wave vector is taken to be $k_{\parallel} = x k_y / L_s$.

In order to study the influence of impurities on the instability, we introduce, besides the main ions (with mass number and charge number 1), a second ion species with $Z$ and $\mu$ as the charge and mass number. From now on, the symbols with subscript $j = i, z$ stand for the main ions (hydrogenic ions) and the impurity ions, respectively. In the fluid treatment here, the ion temperature gradient effect, $\eta_j = d\ln T_j / d\ln n_j$, and the equilibrium radially dependent toroidal velocity $v_{i0}''(x)$ for both the main and impurity ions,

$$v_{0\parallel}(x) = v_{0\parallel}(0) + v_{0\parallel}(0)x$$

are taken into account, where the velocity shear $v_{0\parallel}'$ is a constant. The linear theory of instability driven by parallel velocity shear in plasmas of single-ion-species has been addressed.
The primary modification introduced in this work is the inclusion of the impurity ion species and all corresponding inhomogeneities, especially its flow shear. Linearizing the two-fluid equation, assuming adiabatic electrons, and Fourier transforming in the \( y \) and \( z \) directions, we obtain the eigenmode equation

\[
\frac{d^2 \phi(x)}{dx^2} - b_x \phi(x) + \frac{1 - \tilde{\omega}}{f_x(\tilde{\omega} + K)} \phi(x) + \left( \frac{\varphi_{0||} S}{\tilde{\omega} f_x(\tilde{\omega} + K)} - x + F^2(\tilde{\omega}) \frac{S^2}{\tilde{\omega}^2 x^2} \right) \phi(x) = 0 \tag{1}
\]

where we define the dimensionless parameter "effective velocity shear" as

\[
\varphi_{0||} = (1 - f_x) \bar{\varphi}_{0||} + f_x \tilde{\varphi}_{0||}
\]

and "effective temperature gradient" as

\[
K \equiv \frac{1 - f_x) L_{ei} K_i + f_x L_{ez} K_z \mu / Z^2}{1 - f_x + f_x \mu / Z}
\]

as well as the factors

\[
F(\tilde{\omega}) = \left[ 1 - (1 - Z^2 / \mu^2) \frac{f_x \mu(\tilde{\omega} + L_{ez} K_z / Z)}{f_x Z(\tilde{\omega} + K)} \right]^{1/2}
\]

and

\[
\bar{f}_x \equiv 1 - f_x + \frac{\mu}{Z} f_x.
\]

Here, in Eq. (1) the perturbed electrostatic potential is expressed as \( \tilde{\phi}(x, y, t) = \text{Re}(\phi(x) \exp(ik_y y - i\omega t)) \), where \( x \) is normalized to \( \rho_s \), with \( \rho_s = c_s / \Omega, c_s = (T_e/m_i)^{1/2} \). The ion gyrofrequency is \( \Omega = eB/m_i c, b_x = k^2 \rho_i^2, S = I_{ne} / I_{ni}, \tilde{\omega} = \omega / \omega_{pe} \), and \( \omega_{pe} \) is the electron diamagnetic frequency. \( I_{ne}, I_{ni}, I_{nz} \) are the density gradient scale lengths of electrons, main and impurity ions, respectively, and \( I_{ei} = I_{ne} / I_{ni}, I_{ez} = I_{ne} / I_{nz} \) are the ratios of these scale lengths. Impurity charge concentration is \( f_z = zn_{az} / n_{zc} \). The ratio of impurity ion and main ion mass number is \( \mu = m_z / m_i \). The temperature parameters are \( \tau_i = T_e / T_i, \tau_z = T_e / T_z, \eta_i = I_{ni} / I_{Ti}, \eta_z = I_{nz} / I_{Tz}, K_i = (1 + \eta_i) / \tau_i, K_z = (1 + \eta_z) / \tau_z \). The normalized parallel velocity shears of the main and impurity ions are \( \tilde{\varphi}_{0||} = \varphi_{0||} I_{ne} / c_s \) and \( \tilde{\varphi}_{0||z} = \varphi_{0||z} I_{ne} / c_s \). In addition, the parameters must satisfy the constraint that

\[
L_{ei}(1 - f_x) + L_{ez} f_x = 1 \tag{4}
\]
according to the quasineutrality condition.

In Eq. (1), the "effective velocity shear" defined in Eq. (2) and the "effective temperature gradient" defined in Eq. (3) are used to describe the free energy content induced by ion velocity shear and temperature gradient of both ion species respectively. Eq. (1) is similar to Eq. (7) in Ref. 8. Because of the inclusion of impurity ions, however, significant physical modifications are contained in the effective parameters \( \tilde{v}_{\parallel} \), \( \tilde{K} \), and the factors \( F(\tilde{\omega}) \) and \( \tilde{f}_2 \).

First, the velocity shear takes effect only through \( \tilde{v}_{\parallel} \). It can be seen from Eq. (2) that the contributions from the velocity shears of the main and impurity ions are proportional to the charge density of each species. They enhance each other when they are in the same direction and cancel mutually when in contrast. Hence, the effects of \( \tilde{v}_{\parallel} \) and \( \tilde{v}_{\parallel} \) are shown to be the same qualitatively. But only when \( \tilde{v}_{\parallel} = \tilde{v}_{\parallel} \) and the density and temperature gradients are ignored, the whole effect is quantitatively irrelevant to whether the impurity ions exist or not. On the other hand, the driving effect from the temperature gradient in the \( \tilde{K} \) depends on \( \eta, \eta, L_{ex}, L_{oi} \), as well as the mass and charge number of the impurity ions. Supposing that \( T_i(r) = T_2(r) \), it is necessary that

\[
\eta_k = \eta_k \left( \frac{1}{1 - f_2} \right). \tag{5}
\]

As is shown in the expressions for \( F(\tilde{\omega}) \) and \( \tilde{f}_2 \), the mass effect of the main ion species takes place in the presence of impurity ions. For hydrogen, deuterium and tritium, \( |F_{\tilde{H}}(\tilde{\omega})| < |F_{\tilde{T}}(\tilde{\omega})| < |F_{\tilde{D}}(\tilde{\omega})| \), \( \tilde{f}_{2,H} > \tilde{f}_{2,D} > \tilde{f}_{2,T} \). The appearance of these factors makes it possible to use Eq. (1) for discussing an important topic in the study of fusion plasmas, i.e., the "isotopic scaling" of plasma confinement. (See Figs. 6, 8 and related discussions for detail.)

Equation (1) leads to the dispersion relation

\[
- \frac{\tilde{v}_{\parallel}^2}{4 \tilde{f}_2^2(\tilde{\omega} + \tilde{K})^2} \left[ \frac{1}{\tilde{f}_2(\tilde{\omega} + \tilde{K})} - \frac{\tilde{\omega}^2}{\tilde{f}_2^2(\tilde{\omega} + \tilde{K})} \right] F^2(\tilde{\omega}) - b_2 F^2(\tilde{\omega}) = \frac{i S}{\tilde{\omega}} F^3(\tilde{\omega})(2n + 1). \tag{6}
\]

The corresponding eigenfunction is

\[
\phi^{(n)}(x) = \phi^{(n)}(\tilde{\omega}) F(\tilde{\omega}) H_n \left[ (i S/\tilde{\omega})^{1/2} F^{1/2}(\tilde{\omega}) (x + \Delta) \right] e^{-i S(x + \Delta)^{1/2} F(\tilde{\omega})/2 \tilde{\omega}} \tag{7}
\]

where \( H_n \) is the Hermite function of \( n \)th order, while

\[
\Delta = - \frac{\tilde{v}_{\parallel}^2}{2 S \tilde{f}_2(\tilde{\omega} + \tilde{K}) F(\tilde{\omega})^2}. \tag{8}
\]
In the rest of this work, the \( n = 0 \) mode will be considered only.

There are three driving mechanisms for the eigenmode instabilities in dispersion equation (6): (a) impurity density gradient, (b) parallel velocity shear, and (c) ion temperature gradient. Systematic theoretical investigations have been carried out previously for the correlation between (a) and (c),\(^4\) as well as between (b) and (c).\(^7\) What we mainly discuss here is the correlation between the driving force of impurity ions and parallel velocity shear, together with the case when three of them coexist. In the following discussion, the temperature of impurity and that of the main ions are considered the same,\(^2\) \( T_i(r) = T_\parallel(r), \tau_i = \tau_\parallel = 1. \)

We take \( \eta_i \) and \( L_{ce} \) as the specified parameters, while \( \eta_z \) and \( L_{ei} \) are derived from (4) and (5). We also set \( b_c = k^2 \rho_i^2 \equiv k^2 \rho_i^2 \), and take the carbon fully ionized as the impurity and hydrogen as the main ion unless otherwise stated.

First of all, the influence from the impurity on the driving effect of parallel velocity shear in the absence of ion temperature gradient is studied. Based on the existing analytic result, instability of the impurity mode\(^4,12\) or the parallel velocity shear mode\(^6,13\) may appear even when \( \eta_i = \eta_z = 0. \) The relevant studies, however, did not reveal the interaction of these two mechanisms. Now, we can see from Fig. 1 the mode growth rate and real frequency versus the parallel velocity shear in the presence of impurities. In this figure, results for pure \( (f_z = 0) \) and contaminated \( (\text{the impurity charge concentration } f_z = 0.2) \) plasmas are compared, where the ratio of the density gradient scale lengths \( L_{ce} = -1.1 \), and the velocity shear \( \tilde{v}_u^\parallel = \tilde{v}_u^\parallel_z. \)

It is shown that for a pure plasma the mode growth rate rises when the parallel velocity shear of the main ion increases. In comparison, the presence of the impurity produces a stabilizing effect when its density profile is inwardly peaked, \( L_{ce} > 0 \), whereas it produces a destabilizing effect when its density gradient is oppositely directed from those of the main plasma components, \( L_{ce} < 0 \). However, these effects of impurity density profiles will become less noticeable as \( \tilde{v}_u^\parallel \) increases, and a reverse effect may occur when \( \tilde{v}_u^\parallel \) surpasses a certain limit.

We can see from the definition of \( \tilde{v}_u^\parallel \), Eq. (2), that the impurity concentration does not affect the amplitude of the effective velocity shear when \( \tilde{v}_u^\parallel = \tilde{v}_u^\parallel_z. \) In this case, the
driving mechanism of impurities takes effect only through $\overline{K}$. In Fig. 2, the mode growth rate is shown as a function of the impurity density gradient for $-1 < L_{ez} < 1$ with the concentration $f_z$ fixed. It is shown that the increase of velocity shear always leads to a rise in the mode growth rate. Considering the experimental result that $\overline{\dot{v}_{0||}} \leq 1$, velocity shear is fixed for $\overline{\dot{v}_{0||}} = 0.5$ to discuss the growth rate as a function of the impurity concentration $f_z$ in Fig. 3. It can be seen from Fig. 3 that the increase of impurity concentration enhances the influence of the radial profile of impurity on the instability. The mode growth rate increases (decreases) with the impurity charge concentration linearly for $L_{ne}/L_{nz} < (>) 0$.

The results above demonstrated that the increase of the parallel velocity shear causes the instability to grow, while the driving effect of the impurity is chiefly determined by its radial density distribution. In the range of experimentally relevant values of the parallel velocity shear, $\overline{\dot{v}_{0||}} \leq 1$, when the impurity density gradient is oppositely oriented to that of the electron, the driving forces of the impurity and of the parallel velocity shear enhance each other. Otherwise, the effects of the impurity density gradient tend to cancel the driving mechanism of the parallel velocity shear.

It should be noted that the discussion above is given under the supposition $\overline{\dot{v}_{0||}} = \overline{\dot{v}_{0||}^z}$. Under such conditions, the overall influence of the parallel velocity shear is not changed by the existence of the impurity, and the theoretical analysis with the replacement of $\overline{\dot{v}_{0||}}$ by $\overline{\dot{v}_{0||}^z}$ (experimental measurement) is adequate. However, considering this assumption is probably not satisfied in tokamak plasmas, we fix $\overline{\dot{v}_{0||}^z} \equiv 1$, and let $\overline{\dot{v}_{0||}^z}$ varying from $-1$ to $1$ in Fig. 4, where the mode growth rate changing with $\overline{\dot{v}_{0||}^z}$ is shown. The driving force of the main ion parallel velocity shear is weakened with the increasing of the difference between $\overline{\dot{v}_{0||}^z}$ and $\overline{\dot{v}_{0||}^z}$, $\Delta \overline{\dot{v}_{0||}^z} = |\overline{\dot{v}_{0||}^z} - \overline{\dot{v}_{0||}^z}|$, and the increase of the impurity concentration. In addition, it should be pointed out that even when $\overline{\dot{v}_{0||}^z} = \overline{\dot{v}_{0||}^z}$, the variation of impurity concentration $f_z$ still influences the mode growth rate because the density gradients of the impurity and the majority ions are kept in the previous discussions. In Fig. 4, the growth rates for different $f_z$ will be equal at $\overline{\dot{v}_{0||}^z} = 1$ if we take $\mu = Z$ and $\tau_i, \tau_e \gg 1$ so that $K \approx 0$ and $\overline{\dot{F}}_z = F'(\overline{\dot{v}}) = 1$. 


The influence of ion temperature gradient is shown in Fig. 5 where $\eta_i \neq 0$. The result shows that in the range of $\eta_i$ discussed here the enlargement of the ion temperature gradient accelerates the growth of instability, and makes the driving effect from the impurity even more notable.

In the above calculations we assume hydrogen as the discharge gas. In order to make a further comparison of the mode growth rate in the presence of different hydrogenic isotopes (hydrogen, deuterium, and tritium) as the respective primary ions, the normalized mode growth rate and real frequency as a function of $k_y p_H$ are given in Fig. 6, where $p_H = e\sqrt{2T_i m_H/eB}$ is the hydrogen ion Larmor radius. The mode growth rate is normalized to $\omega_{ci}/k_y p_H$. It can be seen that for a wide range of poloidal wavenumbers (in Fig. 6, $k_y p_H > 0.2 \sim 0.3$) the mode growth rates are $\gamma_H > \gamma_D > \gamma_T$. It is well known that an "isotope scaling" of the plasma energy confinement time $\tau_e \propto M_i^{\alpha}$ conforms to the tokamak experiment, where $M_i$ is the hydrogenic ion mass number and $\alpha = 0.3 \sim 0.7$. The relation of the mode growth rates obtained by the present approach in hydrogen, deuterium and tritium plasmas is physically consistent with the experimental observations. Such relation can hardly be revealed by the conventional linear fluid theory of $\eta_i$ mode. This is due to the combined effect of the three driving mechanisms.

3. Radial ion flux and transport coefficient

The particle transport model as investigated below covers three sources of free energy: the ion temperature gradient, the concentration and radial distribution of the impurity, as well as the parallel velocity shear of both ion species. Until recently, the effect of impurities on anomalous particle transport has been rarely considered in the turbulence transport model. In Ref. 3, the authors mentioned that the profile and concentration of the impurity ions can significantly affect the quasilinear flux induced by drift wave perturbation. However, the influence of parallel velocity shear has not been included despite its importance. The unusual effect of the parallel velocity shear on the plasma confinement as observed in the JT-60U prompts us to do a theoretical analysis on its importance.
Here, the general expression for the perturbed electrostatic potential is

\[ \tilde{\phi} = \sum_{k_y} \phi_0 \phi(x) \exp(-i\omega t + i k_y y). \]

The drift velocity is

\[ \tilde{v}_x = -\frac{c}{B} \frac{\partial \tilde{\phi}}{\partial y}, \quad (9) \]

where \( B \) is the toroidal magnetic field and \( c \) is the speed of light. The amplitude parameter \( \phi_0 \) is used to characterize the RMS fluctuation level and \( \phi(x) \) is the normalized wave function.

Now we introduce the radial microflux function \( \gamma_j \) around the rational surface of the mode,

\[ \gamma_j(x) = \tilde{v}_x^* \tilde{n}_j + \tilde{v}_x \tilde{n}_j^* \quad (10) \]

where * stands for the complex conjugate.

The fluctuation of main ion density is

\[
\tilde{n}_i = (1 - f_{\perp}) \frac{e n_{\perp}}{T_e} \left\{ \frac{L_{ci}}{\omega} - \left( 1 + \frac{L_{ci} K_i}{\omega} \right) \frac{1 - \omega}{f_s (\omega + K)} + \left[ \left( 1 + \frac{L_{ci} K_i}{\omega} \right) \frac{\tilde{v}_0||S}{f_s (\omega + K)} - \frac{\tilde{v}_0||S}{\omega^2} \right] x + (1 - f_\perp^2 (\omega)^2) \left( 1 + \frac{L_{ci} K_i}{\omega} \right) \frac{S^2}{\omega^4} x^2 \right\} \tilde{\phi}_x, \quad (11)
\]

and \( \Gamma_j \) follows from (9) (11).

The microflux \( \gamma_j \) defined by (10) is a function of \( x = r - r_0 \), with \( r_0 \) being the radial position of the mode rational surface. In order to give the effects on the macromotion, \( \gamma_j \) must be integrated over \( x \) around the resonant surfaces \( (x = 0) \) to obtain the corresponding macroflux used in the macroscopic equations on the mode rational surface. Hence we have

\[
\langle \gamma_j \rangle = \int_{-\infty}^{+\infty} \left( \tilde{v}_x^* \tilde{n}_j + \tilde{v}_x \tilde{n}_j^* \right) dx. \quad (12)
\]

In this way, the radial particle flux of the main ion is expressed as

\[
\langle \gamma_j \rangle = \int_{-\infty}^{+\infty} \left( \tilde{v}_x^* \tilde{n}_j + \tilde{v}_x \tilde{n}_j^* \right) dx = -2\sqrt{\pi} |\phi_0|^2 n_{\perp} e \frac{c L_{ci}}{B T_e \rho_x} \Psi(S, K_i, L_{ez}, Z, \mu, f_z, \tilde{\omega}, \tilde{\omega}_0||z), \quad (13)
\]

where the dimensionless function

\[
\Psi(S, K_i, L_{ez}, Z, \mu, f_z, \tilde{\omega}_0||z, \tilde{\omega}_0||z) = \frac{b_j^{1/2} \gamma}{S^{1/2} \Gamma(\omega)^{1/2} \tilde{\omega}} \left\{ -1 + \frac{(1 + K)}{f_s L_{ci} \tilde{\omega} + K} \right\} \frac{K_i (2\omega + K - |\tilde{\omega}|^2)}{f_s |\tilde{\omega} + K|^2}
\]
\[
\frac{\alpha(\tilde{\omega})}{2|\tilde{\omega} + \tilde{K}|^2} \left\{ \tilde{v}_{||}^2 \left[ K_i(\gamma_i^2 - 3\omega_e^2 - 2\omega_e \tilde{K}) - (2\omega_e + \tilde{K})|\tilde{\omega}|^2/L_ei \right] \tilde{f}_z |\tilde{\omega} + \tilde{K}|^2 + 2\tilde{v}_{\|0}^2 \tilde{v}_{||}^2 \omega_e/L_ei \right\} + G(\tilde{\omega}) \right\} \times
\exp \left\{ \frac{\gamma^2 \tilde{v}_{||}^2 \left[ \tilde{K} - (1 - Z^2/\mu^2)L_ei K_z f_z \mu/|Z f_z|^2 \right]^2}{4S \Im(F^*\tilde{\omega})|\tilde{f}_z|^2 |F|^4 |\tilde{\omega} + \tilde{K}|^4} \right\}.
\]

where \( \tilde{\omega} = \omega_e + i\gamma \) is obtained from the mode dispersion equation (6). Meanwhile, in definition (12) we have taken into account the cancellation effect from the opposite contributions of two waves peaking at adjacent rational surfaces on the total radial particle flow in the intervening region, which is consistent with the considerations of the “pairing” principle used by Zhang and Mahajan.\(^{14}\)

In Eq. (14) the term containing \( \alpha(\tilde{\omega}) \{ \cdots \} \) is approximately proportional to the square of the effective velocity shear, and vanishes rapidly when \( \tilde{v}_{\|0} \to 0 \). For a small value of the parallel velocity shear, the magnitude and direction of the flux function is mainly determined by the first and third terms of Eq. (14), which depend on the parameters \( L_ei, \mu, \) and \( f_z \). The factors

\[
\alpha(\tilde{\omega}) = \frac{\Im(F^*\tilde{\omega}) + \tilde{K} \Im(F)}{\tilde{f}_z |F|^2 \Im(F^*\tilde{\omega})}
\]

and

\[
G(\tilde{\omega}) \equiv \left( 1 - \frac{Z^2}{\mu^2} \right) \frac{\mu f_z S|\tilde{\omega}|^2}{\tilde{f}_z |\tilde{\omega} + \tilde{K}|^2} \left( \frac{1}{2 \Im(F^*\tilde{\omega})} + \frac{\tilde{v}_{\|0}^2 |\tilde{\omega}|^2}{4S |\tilde{\omega} + \tilde{K}|^4 \alpha(\tilde{\omega})^2} \right) \left\{ - \frac{2\omega_e + \tilde{K}}{L_ei} \right\}
\]

\[
\frac{\left( K_i + \frac{L_ei K_z}{Z L_e} \right) \gamma_i^2 - 3\omega_e^2 - 2\omega_e \tilde{K}}{|\tilde{\omega}|^2} + \frac{K_i L_ei K_z}{Z |\tilde{\omega}|^4} \left[ 4\omega_e(\gamma_i^2 - \omega_e^2) + \tilde{K}(\gamma_i^2 - 3\omega_e^2) \right] \}
\]

are related to \( Z \) and \( \mu \) of the impurity. When \( Z = \mu \), it turns out that \( \alpha(\tilde{\omega}) \equiv 1 \) and \( G(\tilde{\omega}) \equiv 0 \). As a result, the influence of the last term in Eq. (14) disappears.

From Eq. (12) and the quasineutrality condition, it is easy to obtain the radial impurity particle flux

\[
\langle \Gamma_\rho \rangle = -\frac{1}{Z} \langle \Gamma_\iota \rangle.
\]

The radial particle flow directions of the main ions and impurity ions are opposite. Under the assumption of adiabatic electrons, the driving effect of parallel velocity shear does not lead to a macroflux of the ions in the radial direction.
The effective value of the radial particle transport coefficient of the ions, \( D_j \), is defined as

\[
\langle \gamma_j \rangle = -D_j \nabla_r n_{0j} \tag{16}
\]

where \( \nabla_r \) stands for the radial derivative. Therefore, from Eq. (13), the effective transport coefficient for the main ions is

\[
D_i = -2\sqrt{\pi} |\phi_0|^2 \frac{c e L_{ce}}{B T_e \rho_s} \Psi(S, K_i, L_{cz}, Z_i, \mu_i, f_z, \tilde{\eta}_0, \tilde{\xi}_0, \tilde{\varphi}_0). \tag{17}
\]

The saturation level \( |\phi_0| \) in Eq. (17) can be calculated by different models.

The direction of radial ion flows is of great importance to plasma confinement in fusion studies. It is shown in Eq. (15) that the impurity ions, when included in the ion particle transport, have an opposite flow direction to that of the main ions, while the latter is fully determined by the property of function \( \Psi \). Obviously, it is our hope to see \( \langle \Gamma_i \rangle < 0 \), which means the main ions flow inward. If such a transport behavior can be observed over a wider range of the experimental parameters, it will not only improve the confinement of the main ions but also expel the impurity ions outward. Then, does such a range of parameters exist? And how should we widen this “optimal confinement parameter” range? These are the questions that we now address.

Before describing the parametric dependence in detail, it is appropriate to first observe the change of \( \langle \Gamma_i \rangle \) without the factor \( |\phi_0|^2 n_{0i} c e L_{ce} / B T_e \rho_s \) as a function of \( L_{cz} \). In all subsequent calculations, \( \tilde{\eta}_0 = \tilde{\xi}_0 \) is set. As shown in Fig. 7, where \( \eta_i = 0.5 \) and hydrogen is the discharge gas, \( \langle \Gamma_i \rangle \) changes from positive to negative while \( L_{cz} \) varies from \(-1\) to \(+1\). This inward flow, \( \langle \Gamma_i \rangle \), will increase monotonically with the increase of \( L_{cz} \) after it exceeds a critical value. Meanwhile, it is also found in Fig. 7 that the strengthening of the parallel velocity shear amplifies the inward ion flow \( \langle \Gamma_i \rangle \), while in the case of \( \langle \Gamma_i \rangle > 0 \), the enhancement effect is less apparent.

In order to search for the “optimal confinement parameter” range of \( \langle \Gamma_i \rangle < 0 \), the properties of the dimensionless function \( \Psi \) is investigated. We know that \( \langle \Gamma_i \rangle < 0 \) corresponding to \( \Psi > 0 \), is the condition for an outward impurity flow and inward main ion flow. The turning
point for the sign of \( \langle \Gamma_i \rangle \) is determined by \( \Psi = 0 \). All the physical parameters, including the species of the main ions, exert an influence on this turning point to some extent.

We first analyze the effects of ion temperature gradient. The relation of \( L_{ez} \) versus \( \eta_i \) at \( \Psi = 0 \) is illustrated in Fig. 8. As a comparison, hydrogen, deuterium and tritium are taken as the main ion species respectively. For each kind of main ions, the concentration of the impurity, \( ^{12}\text{C}^6 \) fully ionized, is set at \( f_z = 0.1 \) and 0.2 respectively. In the figure, the area above each critical curve is the corresponding "optimal confinement parameter" range, where the particle fluxes \( \langle \Gamma_i \rangle < 0 \) and \( \langle \Gamma_z \rangle > 0 \). The results clearly show that "optimal confinement parameter" range shrinks with the increase of the ion temperature gradient \( \eta_i \). The reduction of the impurity concentration \( f_z \), on the other hand, is beneficial to widening such range (but unfortunately, as explained later, it reduces the amplitude of \( \langle \Gamma_i \rangle \)). It is also noticeable that the "optimal confinement parameter" ranges of the three main ions, hydrogen, deuterium and tritium are wider in turn when other parameters are fixed. This means that under the same condition, tritium is the first main hydrogen isotope to have an inward particle transport. Deuterium comes second, and hydrogen comes last. This phenomenon may be partly responsible for the "isotope effect" shown in the plasma confinement experiments.

A critical relation governing the particle flow direction within different parameter ranges as well as for different ion species can be derived from Eq. (14). As an example, we analyze the case \( \tilde{\xi}_{\|} \ll 1 \) with deuterium as the main ion. Assuming \( |\tilde{\omega}| \ll \tilde{K} \), the critical parameter relation can be approximated as \( \tilde{K} = \tilde{K} \), i.e.,

\[
L_{ez} = \left[ \left( \frac{Z - 1}{\eta_i} + Z \right) (1 - f_z) + f_z \right]^{-1}. \tag{18}
\]

When \( \eta_i + 1 \ll Z \), we have \( L_{ez} \approx \eta_i/(Z - 1)(1 - f_z) \), in which \( L_{ez} \sim \eta_i \) shows a linear relation.

Now we shift our focus onto the influence of the parallel velocity shear. A critical relation of \( L_{ez} \) versus \( \tilde{\xi}_{\|} \) is plotted in Fig. 9. It can be clearly seen that the increase of \( \tilde{\xi}_{\|} \) expands the "optimal confinement parameter" range substantially. In other words, the parameter range for an inward main ion radial flow becomes wider as a result of the increase of parallel
velocity shear. This velocity shear effect for several choices of $\eta_i$ is also compared. It is found that although the influence of $\hat{v}_{0}^{\parallel}$ is gradually weakened when $\eta_i$ increases, the effect on expanding the parameter range for $\langle \Gamma_i \rangle < 0$ is still strong enough to cancel out the negative influence induced by an increment of the ion temperature gradient.

The dependence of the critical $L_{\perp z}$ on the velocity shear $\hat{v}_{0}^{\parallel}$ for different values of the magnetic shear $S$ is given in Fig. 10. As is shown in the figure, for most of the experimentally relevant parameter regime, the region for $\langle \Gamma_i \rangle < 0$ shrinks when the magnetic shear increases for fixed $b_u$. In addition, the calculation for $b_u = 0.1$ and 0.2 shows that a small poloidal wavenumber corresponds generally to a relatively wide parameter range for $\langle \Gamma_i \rangle < 0$. However, the difference is not significant.

Through the above analysis, we find that while $\eta_i$ and $\hat{v}_{0}^{\parallel}$ are both the driving mechanisms for the turbulence, the increase of $\eta_i$ leads to $\langle \Gamma_i \rangle > 0$, while the increase of $\hat{v}_{0}^{\parallel}$ is just the opposite, $\langle \Gamma_i \rangle < 0$. The increase of $\hat{v}_{0}^{\parallel}$ tends to exert a positive influence over the effective confinement of the main ion as well as the exclusion of impurity ions.

In order to calculate the effective radial particle transport diffusivity coefficient $D_r$, the amplitude of the mode saturation level $|\phi_0|$ needs to be determined. We adopted the estimation method of Ref. 15. Here, the energy density of the unstable mode is assumed to be equivalent to $E_f$, which is the summary of the free energy density contained in the inhomogeneous parallel velocities and the temperature gradients of the main and the impurity ions, i.e.,

$$F_f = \sum_j \left\{ \frac{m_j}{2\Delta} \int_{-\Delta/2}^{\Delta/2} n_{0j} \left[ \nu_{j}^2(x) - \nu_{0j}^2 \right] dx + \frac{3}{2\Delta} \int_{-\Delta/2}^{\Delta/2} \left[ T_j(x) n_j(x) - T_{0j} n_{0j} \right] dx \right\}$$

$$= \sum_j \frac{m_j Z_j^2 v_j^2}{2T_{0j}} |\phi_0|^2 (1 + k^2 \rho_j^2). \quad (19)$$

As an upper limit, the mode saturation level $|\phi_0|$ can be estimated as

$$|\phi_0|^2 = \frac{T_c^2 \rho_c^2}{\nu_c T_{00} E_{\perp z}}, \quad (20)$$

$$E_{\perp z} = \frac{|\omega|^2}{S|F|} \left[ \tau_i (1 - f_i) + \tau_z Z f_z \right]^{-1} \left\{ \frac{1}{3} \left[ (1 - f_z) \hat{v}_{0j}^2 + \frac{\mu}{Z^2 f_z v_j^2} \right] \right\}.$$
Thus, according to Eq. (17), the effective particle transport coefficient of the main ions is

$$D_i = -2 \sqrt{\pi} \frac{e_i \rho_i^2}{L_{ne}} \psi(S, K_i, L_{ee}, Z, \mu, f_z, \bar{v}_0^{||}, \bar{v}_0^{\perp}) \tilde{E}_{iz}.$$  \hspace{1cm} (22)

The radial flux of the main ions is

$$\langle \Gamma_i \rangle = n_{0i} e_i \left( \frac{\rho_i^2}{L_{ni} L_{ne}} \right) A_i$$

$$A_i = -2 \sqrt{\pi} \psi(S, K_i, L_{ee}, Z, \mu, f_z, \bar{v}_0^{||}, \bar{v}_0^{\perp}) \tilde{E}_{iz}.$$  \hspace{1cm} (23)

We can see from Eq. (23) the expression of $\langle \Gamma_i \rangle$ gives a gyro-Bohm scaling with $\langle \Gamma_i \rangle \propto n_{0i} B^{-2}$.

With the following parameter values, $B = 2T$, $T_e = 30$ ev, $L_{ne} = 3cm$, deuterium as the discharge gas, the $D_i$ versus $L_{ne} \lambda_{0i}/c_s dr$ is calculated quantitatively in Fig. 11. The results show that the amplitude of the effective particle transport coefficient increases along with the parallel velocity shear. In parameter ranges leading to an inward impurity flux, the reduction of $\eta_i$ leads to a decrease of the main ion outward flux. In Fig. 12, the effective radial particle transport coefficients of the main (deuterium) ions are given as a function of the impurity concentration $n_{0i}/n_{0e}$ for different types of impurity. The impurities used here, $^{11}$B, $^{12}$C, $^{16}$O and $^{56}$Fe, are assumed to be fully ionized. For lighter impurity ions such as B, C, O, the corresponding transport coefficients of the main ions increase slightly as a result of $Z$ rising. But, the effects of heavy impurity ions such as Fe not only enhance the positive $D_i$ but also lower the absolute value of the negative $D_i$ dramatically. Hence, compared with light impurity ions of the same particle density, the heavy impurity ions are obviously disadvantageous to the confinement of the main ion.

In previous calculations we have set $T_i = T_e = T$, i.e., the same temperature for ions and electron. In order to estimate the effects of the fusion alpha particle product we make a comparison in Fig. 13 for the mode growth rate and corresponding $D_i$ versus $L_{a}$ under the influence of helium ($^{4}$He) impurity ions of concentration $N_{a} = 0.05n_{0e}$ at $T_e = 8T_i$. 

\[ \text{15} \]
and \( T_x = T_i \), retaining \( T_x = T_i \). Specifying \( \eta_i \) and assuming \( dT_x/dx = dT_i/dx \), we have
\[ \eta_i = \eta_i(L_{\text{ex}x} - f_x)\tau_x/(1 - f_x)\tau_i. \]
The simulation result shows a significant difference for the two helium temperatures. With the given parameters, the radially inward peaking of the high temperature helium ion density profile becomes a surprising driving factor for the mode. The variation in the corresponding \( D_i \) versus \( L_{\text{ex}x} \) is also contrary to that of the isothermal helium ions. However, it should be brought to attention that the results of Fig. 13 may not be reliable enough when obtained through a fluid approach, since kinetic effects which are important at high temperature regime are neglected in the fluid theory.

4. Conclusions and discussion

It is widely recognized in theoretical studies and experimental observations that the impurity mode, the \( \nu'_{\parallel} \)-mode and the \( \nu_{\parallel} \)-mode are three principal driving mechanisms for anomalous transport. In this paper, investigation has been made for the first time on the correlation between the impurity mode and the \( \nu'_{\parallel} \) mode. Several results are demonstrated. First, when the impurity species is included, the two flow shears \( \delta \nu'_{\parallel,1} \) and \( \delta \nu'_{\parallel,2} \) take effect through the effective velocity shear \( \overline{\nu'}_{\parallel} \) given in Eq. (2). In \( \overline{\nu'}_{\parallel} \), the signs of \( \delta \nu'_{\parallel,1} \) and \( \delta \nu'_{\parallel,2} \) are the same, indicating the same effect from main ion velocity shear and that of impurities. The quantitative effect is proportional to the ion charge concentration only. Second, the radial profile of the impurity density has a significant influence on the instability of \( \nu'_{\parallel} \) mode. The interplay of these two mechanisms is that, when the radial distribution of the impurity ions is peaking outwardly, i.e., the density gradients of the impurity and the electrons are opposite, the driving force from the impurity gradient and from the parallel flow shear enhance each other. On the contrary, provided the density gradient directions of the impurity ions and the electrons are the same, driving forces of the impurity cancels that of \( \nu'_{\parallel} \) and tends to exert a stabilizing effect on the \( \nu'_{\parallel} \) mode. Furthermore, this work gives a comprehensive consideration of the parametric dependence of the mode growth rate under the simultaneous influence of all the three principal driving forces.

The particle transport from turbulence driven by parallel velocity shear, ion temperature
gradient and impurity density gradient is investigated. The particle flux obtained in this work has the scaling \( \langle I_i \rangle \sim n_0 B^{-2} \) which is in agreement with the experimental observations.\(^9\) The estimated magnitude of the particle diffusion coefficient shown in Fig. 11 is in the range experimentally observed in tokamak plasmas.

More noticeable is the direction of the particle flux. It is illustrated analytically that the directions of the main ion and the impurity ion fluxes change directions provided \( L_{cz} \) and \( \eta_i \) satisfy certain relations. Over a relatively wide range of parameters, when \( L_{cz} \) is smaller than a certain positive value, the inward impurity particle flow and outward main particle flow will occur. In contrast, when \( L_{cz} \) is greater than this critical value, there will be an inward main ion particle motion with an outward impurity ion flow in the same plasma region, corresponding to the 'anomalous pinch' effect of the main ions. The occurrence of such an effect provides the opportunity to improve the particle confinement of the main ions and to expel the impurity ions. To prevent the accumulation of impurities it is necessary to have a significant fusion power \( Q \)-value. The particle fluxes are obtained as a function of the ion temperature gradient and the impurity density profile. An inward flow of impurity ions and an anomalous main ion particle pinch are both possible. Therefore a theoretical explanation can be given for these two striking experimental phenomena within the framework presented here. It has recently been suggested that the reduction of ion heat flow is connected to properties of the impurity ion profile,\(^11\) and the reduction in electron thermal diffusivity may correlate to reduced \( \eta_e \). Similarly, as shown in this work, the decrease of the outward ion mass flow depends on the impurity distribution and \( \eta_i \). An important result is obtained from the data shown in Figs. 9 and 10. The increase of the parallel velocity shear can effectively widen the range for a inward mass flow of the main ions, which consequently brings a dramatic improvement to the plasma confinement. To the opposite, the increase of \( \eta_i \) shrinks the favorable parameter range. The experimental results from JT-60U indicate that, in the region with a very strong \( dv/dr \), the confinement property of plasmas is improved dramatically. This fact serves to reinforce the theoretical results of this work.

It should be clear that the anomalous particle transport in tokamak plasmas is rather
complicated. The present work is a first study on this important subject, and, in order not to make it too complicated, essential factors such as toroidicity are not included here. In toroidal geometry, two additional mechanisms appear: toroidal (curvature and magnetic gradient) drift and linear coupling of neighbouring harmonics of the modes. Fortunately, it has been shown$^{16}$ that these mechanisms do not lead to dramatic changes to the instability properties of the modes. We expect the same conclusion to apply for the transport properties studied in the present work. Nevertheless, toroidal effects have to be considered in future works.

In addition, estimation of the turbulence saturation level is a big issue that has not been solved up to now. A complete and self-consistent solution to the problem is a challenge faced by physicists. The saturation level estimated in this work is preliminary. It does not include the stabilization effects from perpendicular velocity shear and thus may be overestimated. Nevertheless, the basic gyro-Bhoom scaling revealed in Eq. (22), $D_i \propto c_s \rho_s^2$ is in agreement with the recent gyro-kinetic simulation results.$^{17}$ $\chi_i \propto c_s \rho_s^2$.

Furthermore, restraint effects induced by collisional damping mechanics are not taken into account in this quasilinear framework.

The main trends reported, however, are expected to be dominant and correlate with applicable computer simulations and experiments.

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References


FIG. 1. Normalized mode growth rate vs parallel velocity shear for $\dot{\gamma}_{0/1} = \dot{\gamma}_{0/z}, f_z = 0.2$ and 0, $L_{ez} = -1$ and 1. The other parameters are $b_1 = 0.1, \zeta = 0.1, \eta_i = 0$, hydrogen as the main ion and carbon fully ionized as the impurity ion.

FIG. 2. Normalized mode growth rate vs impurity distribution $L_{ez}$ for $\dot{\gamma}_{0/1} = \dot{\gamma}_{0/z} = 0.5, 1.0, 1.35$ and 1.5, $f_z = 0.2$, The other parameters are the same as fig. 1.
FIG. 3. Normalized mode growth rate vs impurity concentration \( f_z \) for \( L_{eq} = -1, -0.5, 0.5, 1 \). The other parameters are \( \nu = 0.5, b_s = 0.2, f_z = 0.2, \eta_i = 0, s = 0.1 \), hydrogen as the main ion and Carbon fully ionized as the impurity ion.

FIG. 4. Normalized mode growth rate vs the impurity velocity shear \( \nu_{0//z} \) for \( f_z = 0.01, 0.1, 0.2, 0.4 \), and \( \nu_{0//i} = 1.0 \) fixed. The other parameters are \( s = 0.1, b_s = 0.1, L_{eq} = 1, \eta_i = 0 \), with the same ions as in fig. 3.
FIG. 5. Normalized mode growth rate vs ion temperature gradient $\eta_0$ for $L_{ez} = -1$ and 1, $\nu_0/\nu_i = \nu_{0/z} = 0.5$ and 1. The other parameters are $s=0.1$, $b_s=0.2$, $f_z=0.2$, carbon fully ionized as the impurity and hydrogen as the main ion.

FIG. 6. Normalized mode growth rate and the real frequency vs the poloidal mode number for hydrogenic plasmas, i.e. hydrogen, deuterium and tritium, with carbon fully ionized as the impurity. The other parameters are $\nu_0/\nu_i = \nu_{0/z} = 0.3$, $f_z=0.2$, $s=0.2$, $L_{ez}=0.5$ and $\eta_i = 3$. 

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FIG. 7. Particle flux of main ion $\langle f_i \rangle$ without the factor $|\hat{q}_0|^2 n_{oi} e c L_{ei}/B T e^2 \rho_s$ vs the impurity ion distribution $L_{ne}/L_{nz}$ for $\tilde{v}_{0/1}=0, 0.5, 1$ and 1.5, with the main ion hydrogen and the impurity Carbon fully ionized. The other parameters are $\tilde{s}=0.1, b_s=0.1, \eta_i=0.5, \xi_z=0.2$.

FIG. 8. $L_{ez} \sim \eta_i$ critical relation of $\Psi=0$ for $\xi_z=0.1$ and 0.2, with the main ions hydrogen, deuterium and tritium respectively. The other parameters are $\tilde{v}_{0/1}=0.5, s=0.1, b_s=0.1, Z=6$ (carbon).
FIG. 9. $L_{ez} \sim \varphi_{0/1}^i$ critical relation for $\Psi = 0$ with the main ion deuterium, and $\eta_1=0.5, 1.0, 1.5$.  
2. The other parameters are $s=0.2$, $b_1 = 0.2$, $Z=6$ (carbon), $f_2 = 0.1$. 

FIG. 10. $L_{ez} \sim \varphi_{0/1}^i$ critical relation for $\Psi = 0$ with the main ion deuterium, and $s=0.1, 0.2, 0.3$ and 0.4 for $b_1 = 0.2$; $s=0.2$ for $b_1 = 0.1$. The other parameters are $\eta_1 = 0.5$, $f_2 = 0.1$, $Z=6$ (carbon).
FIG. 11. Particle transport coefficient of the main ion vs parallel velocity shear for $\eta_i=0.1, 0.5$ and $1.0$, $L_{az}=-1$ and 1. The other parameters are $s=0.1$, $b_z=0.1$, $B=2$ T, $T_e=30$ ev, $L_{ne}=3$ cm, $f_z=0.2$, $Z=6$ (carbon), deuterium as the main ion.

FIG. 12. Particle transport coefficient of the main ion vs the impurity density $n_{oz}/n_{oe}$ for $L_{az}=-1$ and 1, $\eta_i = 1$, $\dot{\gamma}_{0/1} = 0.5$. The other parameters are the same as fig. 11.
FIG. 13. Normalized mode growth rate and $D_i$ vs the impurity profile $L_{se}$ effected by impurity helium with $T_z=8T_i$ (high temperature) and $T_z=T_i$. The other parameters are $\tau_i=1.0, \theta_i=0.1, n_{0z}=0.005 n_{0e}, T_{0e}/T_i=0.5, n_i=1.5, L_{se}=1$, deuterium as the main ion.