THE KAON ELECTROMAGNETIC FORM FACTOR AND EFFECTS OF RUNNING COUPLING CONSTANT

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ABSTRACT

The Borel transform and resummed expression for the kaon electromagnetic form factor $F_K(Q^2)$ are obtained in the context of QCD running coupling $\alpha_S(Q^2(1-x)(1-y))$ approach. It is demonstrated that effects of running coupling (infrared renormalons) can be taken into account by scale-setting procedure $\alpha_S(Q^2) \rightarrow \alpha_S(e^{\Gamma(Q^2)}Q^2)$ in the leading order expression.

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1. One of the important problems of the perturbative QCD (pQCD) is an investigation of the infrared renormalon effects in the various inclusive and exclusive processes. Their resummation and evaluation corresponding power suppressed corrections to processes' characteristics\(^1\)\(^2\). It is well known that infrared renormalons are responsible for factorial growth of coefficients in perturbative series for the physical quantities\(^3\). But these large contributions can be resummed by means of the Borel transform into the scale $\hat{Q}^2$ used in the coupling constant $\alpha_S(\hat{Q}^2)$ at the one-loop order. Technically, all-order resummation of infrared renormalon contributions correspond to the calculation of the one-loop Feynman diagrams with the running coupling constant $\alpha_S(-k^2)$ at the vertices, where $k$ is the momentum flowing through the virtual gluon line. The calculation of the Feynman diagrams with the running coupling $\alpha_S(-k^2)$ is a generalization of the Brodsky, Lepage and Mackenzie (BLM) scale-setting prescription\(^4\), which amounts to absorbing certain vacuum polarization corrections appearing at higher-order into the one-loop QCD coupling constant.

The BLM scale setting procedure is applicable also for exclusive processes. But here, as it has been demonstrated in Ref.[2], there is an additional source of infrared renormalon contributions in exclusive processes, namely, running coupling constant $\alpha_S(Q^2xy)$, where $x$ and $y$ are longitudinal fractional momenta of hadron constituents. In the case of the pion electromagnetic (elm) form factor these contributions have been resummed into the scale of $\alpha_S(\hat{Q}^2)$ at the leading order result\(^2\).

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2. In our previous letter\(^3\) (hereafter I) we have calculated the kaon \(eN\) form factor \(F_k(Q^2)\) in the running coupling approach. For regularization of infrared singularities we have applied the principal value method\(^3,6\) and obtained for \(F_k(Q^2)\) the perturbative series in \(Q^2\). In this work using the expressions for \(F_k(Q^2)\) found in I, we shall prove that contributions of the infrared renormalons to \(F_k(Q^2)\) can also be resummed into the scale \(\alpha_s(Q^2)\).

In our calculations of \(F_k(Q^2)\) in I the two methods have been used. In the context of the first method we have applied the expression

\[
J_{\alpha} = \frac{1}{t} \int \frac{dw}{w} \frac{(2x-1)^\alpha(2y-1)^\alpha}{t + \ln(1-x) + \ln(1-y)} \sum_{n=0}^\infty \sum_{k=0}^n \frac{n!}{k!} c^{n-k} c^k,
\]

(1)
directly to Eqs.(I, 9a,9b) (explicit expressions for \(C_1\) can be found in Eq.(1,32)) and when the dependence of the kaon wave function \(\phi_k(x,Q^2)\) on the scale \(Q^2\) was ignored, we obtained

\[
Q^2 F_k(Q^2) = (80\pi f_k)^2 \sum_{n=1}^\infty \frac{\alpha_s(Q^2)}{4\pi} \beta_0^{-1} R_n.
\]

(2)

In Eq.(2) \(R_n\) has the form

\[
R_n = (n-1)! \sum_{k=1}^\infty \frac{n m_k}{k^{n+1}},
\]

(3)

where the coefficients \(m_k, n_k\) depend on the kaon wave function constants \(a, b, c\) (Eq.(I,10))

\[
m_1 = (a+b)^2 + c^2 + \frac{2}{3}c(a+b), \quad m_2 = (a+5b)^2 + 49c^2 + \frac{14}{3}c(a+5b),
\]

\[
m_3 = 64b^2 + 324c^2 + 96bc, \quad m_4 = 16b^2 + 400c^2 + \frac{160}{3}bc, \quad m_5 = 64c^2,
\]

(4a)

and

\[
a_1 = -2(a+b)(a+\frac{7}{3}b) - \frac{16}{3}c^2 - \frac{2}{3}(11a/3 + 5b),
\]

\[
a_2 = 2(a^2 - 25b^2) - \frac{406}{3}c^2 - \frac{2}{3}c(8a+250b),
\]

\[
a_3 = 8b(c+b) - 342c^2 - \frac{2}{3}c(67b-9a),
\]

\[
a_4 = \frac{4}{3}b(35e-a) + \frac{820}{3}c^2 + \frac{2}{3}c(144b - 10a/3),
\]

\[
a_5 = 628c^2 + \frac{4}{3}c(a+17b).
\]

(4b)

As it is seen from Eq.(3) \(R_n\) are proportional to \((n-1)!\). Therefore the series is ill-defined and must be resummed by using the method of the Borel transform for removing divergences.

The Borel transform of the series (2) is defined as

\[
B[Q^2 F](u) = \sum_{n=1}^\infty \frac{u^{n-1}}{(n-1)!} R_n.
\]

(5)

Eq.(5) can be inverted to give the resummed expression for \(Q^2 F_k(Q^2)\)

\[
(Q^2 F_k(Q^2))^{\text{res}} = \sum_{n=1}^\infty \frac{u^{n-1}}{(n-1)!} R_n.
\]

(6)

The infrared renormalons appear in Eq.(6) as poles of the integrand on the positive real \(u\) axis. Therefore, for calculation of \((Q^2 F_k(Q^2))^{\text{res}}\) a some regularization method, for example, the principal value prescription, is needed.

For calculation of \((Q^2 F_k(Q^2))^{\text{res}}\) we can follow this general recipe, but the second method used in I (the so-called "integral representation" approach) allows us to get directly both the Borel transform \(B[Q^2 F](u)\) of the series (2) and the resummed kaon \(eN\) form factor \((Q^2 F_k(Q^2))^{\text{res}}\). Indeed, after minor modification in Eq.(I,11)

\[
\int \frac{e^{(t+w+z)}}{s} ds = \frac{1}{t+w+z},
\]

namely, changing \(s\rightarrow u\) and using the value of the \(t, 1/t=4\pi/\beta_0\alpha_s(Q^2)\)
one can immediately find the Borel transform of Eq. (2). In fact, the integrand in Eq. (1.14) is the Borel transform of the term $I_k^k$. The Borel transform of the whole series (2) is given by the expression

$$B(Q^2F)(u) = \sum_{k=1}^{\infty} \left( \frac{m_k}{(k-u)^2} + \frac{n_k}{k-u} \right).$$

The series (2) can be recovered by applying

$$R_n = \left( \frac{d}{du} B(Q^2F) \right)_{u=0}.$$  

The $B(Q^2F)(u)$ has double and single poles at $u=k$ ($k=1;2;3;4;5$). Then the resummed expression $[Q^2F_k(Q^2)]^{res}$ can be calculated by help of the principal value method. The Eqs. (1.20) are, in fact, such resummed expressions for $I_k^k$. It is convenient to rewrite $[Q^2F_k(Q^2)]^{res}$ in the following form

$$[Q^2F_k(Q^2)]^{res} = \frac{(8\pi\alpha_{em})^2}{\beta_0} \sum_{n=1}^{\infty} \left[ \frac{m_k}{k} + (n_k + m_k \ln \lambda) \frac{Li(\lambda^2)}{\lambda^2} \right].$$

where, $Li(\lambda)$ is the logarithmic integral

$$Li(\lambda) = \int_0^1 \frac{dx}{\ln x}, \quad \lambda = \frac{Q^2}{\Lambda^2}.$$  

3. The principal value prescription used above to regulate the infrared renormalon singularities in the resummed expression $[Q^2F_k(Q^2)]^{res}$ (see, Eq. (6)) produces an ambiguity in the perturbative series for the kaon form factor. Any other approach for defining the integral (6) would differ in the treatment of these singularities. The ambiguity introduced by our treatment, which can be evaluated as in Ref. [2], is a higher twist and for $Q^2F_k(Q^2)$ is in order of $\lambda^2/Q^2$. This ambiguity, probably, may be combined with a contribution from the kaon's higher Fock state wave function $|q\bar{q}'g\rangle$. Indeed, there is an additional virtual gluon propagator in the Feynman diagrams of the subprocess $q\bar{q}'g\gamma^* - q\bar{q}'g$; their $1/Q^2$ suppression relative to the leading ones is caused by this hard gluon propagator.

The higher twist correction may modify Eq. (8) at $1/\Lambda$ level

$$\left( (n_{1+}, m_{1+} \ln \lambda) Li(\lambda) \right) /\lambda \rightarrow \left( (n_{1+}, m_{1+} \ln \lambda) Li(\lambda) + a/\lambda.\right.$$  

For clarifying this problem further investigation is needed.

4. For studying phenomenological consequences of the infrared renormalon corrections to $Q^2F_k(Q^2)$ it is useful to introduce a ratio $R=[Q^2F_k(Q^2)]^{res}/[Q^2F_k(Q^2)]^0$, where $[Q^2F_k(Q^2)]^0$ is the kaon leading order form factor, calculated in the context of the ordinary ("frozen coupling") approach

$$[Q^2F_k(Q^2)]^0 = 1600\pi^2\alpha_s(Q^2) \left[ \frac{a}{2} - \frac{b}{5} \right]^2 + \frac{c}{10} \left[ \frac{a}{3} - \frac{b}{5} + \frac{c}{10} \right]^2.$$  

In Fig. 1 the dependence of the ratio $R$ on $Q^2$ is shown. It is evident that the renormalon corrections are considerable for all values of $Q^2$. Thus, $R$ grows from 1.62 to 2.55 when $Q^2$ changes from 2 GeV$^2$ to 14 GeV$^2$, decreasing up to 2.48 at 30 GeV$^2$ (is not shown in Fig. 1).
The infrared renormalon contributions can be transferred into the scale of $\alpha_s(Q^2)$ in Eq. (10). In Ref.[4] in the next-to-leading order scale-setting calculations the following redefinition of the scale $Q^2$ was proposed

$$Q^2 \rightarrow e^{f(Q^2)}Q^2,$$

where

$$f(Q^2)=c_1+c_2\alpha_s(Q^2).$$

In our previous work (see, Ref.[2]) the scale-setting procedure (see, Eqs.(11),(12)) have been used. Numerical fitting shows, however, that for the kaon a more reliable form of $f(Q^2)$ is

$$f(Q) = 0.3346 - 59.232 <X_{S}(Q^2) + 200.785 \left[\alpha_s(Q^2)\right]^2$$

It is important to note that the function $f(Q^2)$ (Eq.(13)) cannot be found directly from the perturbative series (2) by applying the BLM next-to-leading order scale-setting procedure.

The ratio $R$ with the redefined scale $Q^2 \rightarrow e^{f(Q^2)}Q^2$ in the leading order expression (10) is shown in Fig.1. As it is seen from Fig.1, in the whole range of $Q^2$ the ratio $R=1$.

The redefinition of the scale $Q^2 \rightarrow e^{f(Q^2)}Q^2$ means, that an average virtuality of the exchanged gluon in the kaon form factor diagrams is taken equal to $e^{f(Q^2)}$. The coupling constant $\alpha_s(e^{f(Q^2)}Q^2)$ varies from $\alpha_s=0.42$ at $Q^2=2$ GeV$^2$ to $\alpha_s=0.41$ at $Q^2=20$ GeV$^2$, reaching its maximum value $\alpha_s=0.52$ at $Q^2=7$ GeV$^2$. These values are typical for such kind of studies.

In Fig.2 the dependence of the kaon form factor on $Q^2$ is shown. From Fig.2 it follows that there is difference between the resummed and the "bare" kaon form factors. This difference indicates the importance of the running coupling effects in exclusive processes.

5. In this letter we have found the Borel transform of the perturbative series and resummed expression for $Q^2F_k(Q^2)$. We have demonstrated that the Borel transform of the series (2),(3) suffers from infrared renormalon singularities, which can be regularized by applying the principal value prescription. The obtained infrared induced corrections have been taken into account by redefining the scale $Q^2 \rightarrow e^{f(Q^2)}Q^2$ of $\alpha_s(Q^2)$ in the leading order expression (10).

It is worth noting that a lucky guess of the integral representation, which allowed us to get directly the Borel transform of the series (2), is valid only when the kaon wave function $\Phi_k(x,\hat{Q}^2)$ is independent on the scale $\hat{Q}^2$ ($\Phi_k(x,\hat{Q}^2)=\Phi_k(x,\mu)$). Otherwise, one must perform calculations in accordance with "the full scheme" described above, i.e. one has to calculate a perturbative series similar to Eq.(2), then to reconstruct its Borel transform and finally to obtain a resummed expression for $Q^2F_k(Q^2)$.

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REFERENCES


FIGURE CAPTIONS

Fig.1. The ratio R is shown as a function of $Q^2$. Here the curves 1 and 2 correspond to $R$ with $(Q^2 F_K(Q^2))^0$ in the frozen coupling approximation (Eq.(10)) and after the scale-setting procedure $Q^2 \rightarrow e^{Q^2/Q_F^2}$, respectively. In calculations the kaon wave function's parameters have been taken to be equal to $a=0.08$, $b=0.6$, $c=0.25$ and $A=0.1$ GeV.

Fig.2. The dependence of the kaon elm form factor on $Q^2$. The curve 1 corresponds to $Q^2 F_K(Q^2)$ found after the scale-setting procedure, whereas the curve 2 describes it in the framework of the ordinary approach.