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PINNING OF CURVED FLUX LINES ON THE BOUNDARY BETWEEN TWO PHASES OF HIGH TEMPERATURE SUPERCONDUCTORS *

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ABSTRACT

The distribution of the magnetic fields and the current density in a system of two phases of high temperature superconductors (HTS) are determined. Recently, this system has been confirmed experimentally [1,2]. The energy of the interaction between curved flux lines (FLs) and the interface is calculated. It is shown that for positive ΔG (the change in the flux line (FL) energy at the interface) the FLs will be pinned by the interface and the pinning force is formulated. Moreover, this force will be different according to the movement of the FL to the right or to the left. Agreements with the previous results (in special cases) are obtained. This study may serve to test the pinning experimentally.

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1. INTRODUCTION

It is well known that the critical currents j_c in the superconductors are of basic interest. This is because of its origin (flux jump, thermal instabilities, pair-breaking, etc.) and for its dependence on various parameters (temperature, magnetic field, voltage, etc.). Moreover the enhancement of the critical currents are due to the pinning forces.

It is now established that annealing $Bi_4Ca_3Sr_3Cu_4O_{16}$ in flowing oxygen at $\sim 885^\circ C$ results in a material with two superconducting transitions $T_{c1} \sim 85$ K and $T_{c2} \sim 115$ K (i.e. the sample is composed of two phases, with different T_c's) [1]. The analysis of magnetization data by Alexander [1] suggested that such materials consist of two kinds of regions in contact, each having a T_c of 85 K or 115 K. Moreover, the presence of two phases with $T_{c1} \sim 75$ K and $T_{c2} \sim 104$ K has been confirmed, in the ceramic compound $Bi_1Ca_1Sr_{0.7}Al_{0.5}Cu_4O_y$ by Aleksandrov [2]. Thus it is interesting to study the pinning of FLs on the boundary between two phases of HTS, which will be considered in this work.

2. TWO PHASES MODEL

Consider a system of two phases of HTS with two superconducting transitions T_{c1} and T_{c2} ($T_{c1} < T_{c2}$) in an external applied magnetic field $\vec{B}_a$. The two phases having penetration depths λ_1, λ_2 and coherence lengths ξ_1, ξ_2, respectively (with $\lambda_1 > \xi_1$, $\lambda_2 >> \xi_2$). A planar interface of the two phases will be located at $x = 0$ with $x > 0$ is the first phase and $x < 0$ is the second phase. At $T < T_{c1}$, the two phases coexist. In the London limits, $B_{c1} \leq B_a < B_{c2}$ (B_{c1} - the first critical field and B_{c2} - the second critical field), the magnetic field penetrates into the sample in the form of a distorted - or straight - FL (here, we will consider the case of distorted FLs). In the case of $\lambda_1 > \lambda_2$, the total magnetic fields can be written in the form

$$\begin{aligned} \vec{B}_1 &= \vec{B}_{L1} + \vec{B}_\nu + \vec{B}_{S1} \qquad , \quad x > 0 \\ \vec{B}_2 &= \vec{B}_{L2} + \vec{B}_{S2} \qquad , \quad x < 0 \end{aligned} \tag{2.1}$$

where $\vec{B}_\nu$ is the FL fields which obey the following equation:

$$-\lambda^2 \nabla^2 \vec{B} + \vec{B} = \phi_0 \sum_\nu \left[\oint d\vec{r}_\nu \, \delta(\vec{r} - \vec{r}_\nu) - \oint d\vec{r}_{\nu 1} \, \delta(\vec{r} - \vec{r}_{\nu 1}) \right]. \tag{2.2}$$

Here $\vec{r} \equiv (x,y,z)$, $\vec{r}_\gamma \equiv (x_\nu(z), y_\nu(z), z)$ is the locus of the ν^{th} FL singularity and its image in the opposite direction is given by $\vec{r}_{\nu 1} \equiv (-x_\nu(z), y_\nu(z), z)$. $\vec{B}_{L1}$ and $\vec{B}_{L2}$ are the London penetrating fields.

$\vec{B}_{S1}$ and $\vec{B}_{S2}$ are the stray fields. Both of these fields $(\vec{B}_L, \vec{B}_S)$ satisfy the homogeneous of Eq.(2). The appropriate boundary conditions at the interface $x = 0$ are

$$B_{1t} = B_{2t}$$
$$\lambda_1^2 (\mathrm{curl}\, \vec{B}_1)_t = \lambda_2^2 (\mathrm{curl}\, \vec{B}_2)_t \qquad (2.3)$$

the suffix t indicates the tangential component.

One can find

$$\vec{B}_{L1} = \vec{a} \exp(-x/\lambda_1) \,,$$
$$\vec{B}_{L2} = \vec{b} \exp(+x/\lambda_2) \,. \qquad (2.4)$$

$\vec{B}_v$ can be found [4]

$$\vec{B}_v = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \vec{B}_v(\vec{k})$$
$$= \frac{\phi_0}{4\pi\lambda_1^2} \sum_\nu \left[\oint d\vec{r}_\nu \frac{e^{-|\vec{r}-\vec{r}_\nu|/\lambda_1}}{|\vec{r}-\vec{r}_\nu|} - \oint d\vec{r}_{\nu_1} \frac{e^{-|\vec{r}-\vec{r}_{\nu_1}|/\lambda_1}}{|\vec{r}-\vec{r}_{\nu_1}|} \right] , \qquad (2.5)$$

where $\vec{B}_v(\vec{k})$ is given by [4]

$$\vec{B}_v(\vec{k}) = \frac{\phi_0}{1+k^2\lambda_1^2} \sum_\nu \left[\oint d\vec{r}_\nu e^{-i\vec{k}\cdot\vec{r}_\nu} - \oint d\vec{r}_{\nu_1} e^{-i\vec{k}\cdot\vec{r}_{\nu_1}} \right]$$
$$= 2 \frac{\phi_0}{1+k^2\lambda_1^2} \sum_\nu \int dz\, e^{-i\vec{k}_2\cdot\vec{r}_\nu} \begin{pmatrix} x'_\nu \cos(k_x x_\nu) \\ -y'_\nu i \sin(k_x x_\nu) \\ -i \sin(k_x x_\nu) \end{pmatrix} \qquad (2.6)$$

$\vec{k}_2 \equiv (0, k_y, k_z)$, $x' = \frac{dx}{dz}$, $y' = \frac{dy}{dz}$, $k = |\vec{k}|$ and $k_2 = |\vec{k}_2|$

It is easy to show that $B_{vy} = B_{vz} = 0$ at the interface.

The stray fields can be found in the form

$$\begin{matrix}\vec{B}_{S1} \\ \vec{B}_{S2}\end{matrix} = -\int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{r}} \frac{B_{vx}(\vec{k})}{k_2(k_2+\alpha)} \begin{cases} (k_2^2 \hat{x} - i\vec{k}_2 \alpha)\, e^{-\alpha x} \\ (k_2 \alpha \hat{x} + i\vec{k}_2 \alpha)\, e^{\beta x} \end{cases} \qquad (2.7)$$

where $\alpha = (\lambda_1^{-2} + k_2^2)^{1/2}$ and $\beta = (\lambda_2^{-2} + k_2^2)^{1/2}$ are the reciprocal penetration depths of the stray field into the two phases, respectively.

The total current density inside the two phases $\vec{J}_1$ and $\vec{J}_2$ ($\vec{J} = \frac{c}{4\pi}$ curl $\vec{B}$) can be found

$$\vec{J}_1 = \frac{c}{4\pi} \Big[\frac{e^{-x/\lambda_1}}{\lambda_1} (\hat{y} a_z - \hat{z} a_y) + \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} i\vec{k} \wedge \vec{B}_v(\vec{k}) +$$
$$+ \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{r} - \alpha x} B_{vx}(\vec{k}) \left(\frac{\alpha}{k_2} - 1\right) i (\hat{y} k_z - \hat{z} k_y) \Big] , \qquad (2.8)$$

$$\vec{J}_2 = \frac{c}{4\pi} \Big[- \frac{e^{x/\lambda_2}}{\lambda_2} (\hat{y} b_z - \hat{z} b_y) + \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{r} + \beta x} B_{vx}(\vec{k}) \,\cdot$$
$$\cdot \frac{\alpha}{k_2} \frac{\left(\frac{\beta}{k_2} - 1\right)}{\left(\frac{\alpha}{k_2} + 1\right)} i (\hat{y} k_z - \hat{z} k_y) \Big] . \qquad (2.9)$$

By using the boundary conditions (3), we get

$$\vec{a}_2 = \frac{1}{(\sigma+1)} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{r}} i \Big[\sigma k_2 \lambda_1 \vec{B}_{v2}(\vec{k}) -$$
$$- \frac{\alpha}{k_2} \lambda_1 B_{vx}(\vec{k}) \Big\{ \sigma + \frac{1}{(k_2+\alpha)} \Big(\frac{2}{\lambda_1} + \frac{k_2 - \beta}{\sigma} \Big) \Big\} \vec{k}_2 \Big] , \qquad (2.10)$$

$$\vec{b}_2 = \frac{\sigma}{(\sigma+1)} \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}_2\cdot\vec{r}} i \Big[k_y \lambda_1 \vec{B}_{v2}(\vec{k}) -$$
$$- \frac{\alpha}{k_2} \lambda_1 B_{vx}(\vec{k}) \Big\{ 1 - \frac{1}{(k_2+\alpha)} \Big(\frac{2}{\lambda_1} - \frac{k_2 - \beta}{\sigma^2} \Big) \Big\} \vec{k}_2 \Big] . \qquad (2.11)$$

with $\vec{a}_2 \equiv (a_y, a_z)$, $\vec{b}_2 \equiv (b_y, b_z)$, $\vec{B}_{v2}(\vec{k}) \equiv (B_{vy}(\vec{k}), B_{vz}(\vec{k}))$,

and $\sigma = \lambda_1/\lambda_2$.

It is easy to show that $[\vec{k} \wedge \vec{B}_v(\vec{k})]_x = 0$, this means that $\vec{a}$ and $\vec{b}$ are independent of $\vec{r} \equiv (x,y,z)$.

Actually, Eq.(2.3) indicates that the superconducting current is refracted by the interface, which leads to the interaction of the FLs with the boundary between the two HTS phases. Also, the transport current can flow along the boundary in a layer of thickness $\sim (\lambda_1 + \lambda_2)$.

3. FREE ENERGY

The Gibbs free energy of the system can be written in the form

$$G = \frac{1}{2\mu_0}\int d^3r\,[\vec{B}_1^{\,2} + \lambda_1^2(\mathrm{curl}\,\vec{B}_1)^2] + \\ + \frac{1}{2\mu_0}\int d^3r\,[\vec{B}_2^{\,2} + \lambda_2^2(\mathrm{curl}\,\vec{B}_2)^2]\,. \tag{3.1}$$

Due to the linearity of both of the Maxwell equations ($\vec{J} = \frac{c}{4\pi}\,\mathrm{curl}\,\vec{B}$) and the London equation (2) the magnetic field is the linear superposition of contributions of all FL elements. For $\lambda_1 > |\vec{r}_\nu| > \lambda_2$, we can express the Gibbs free energy as follows:

$$G_1 = \frac{1}{2}\sum_\nu \oint dr_\nu \sum_\mu \Big[\oint dr_\mu\, V_3(|\vec{r}_\nu - \vec{r}_\mu| - \oint dr_{\mu_1} V_3(|\vec{r}_\nu - \vec{r}_{\mu_1}|)\Big] + \\ + \sum_\nu \int dz \sum_\mu \int dz'\, x'_\nu(z')\, V_S(\vec{r}_\nu(z), \vec{r}_\mu(z')) - \\ - \frac{1}{\mu_0} S(\lambda_1|\vec{a}|^2 + \lambda_2|\vec{b}|^2) \quad , \lambda_1 > |\vec{r}_\nu| > \lambda_2 \tag{3.2}$$

Here, we use the identity

$$\int_{x \gtrless 0} d^3r\,(\mathrm{curl}\,\vec{B})^2 = -\int_{x \gtrless 0} d^3r\,\vec{B}\;\mathrm{curl}\,\mathrm{curl}\,\vec{B} + \\ + \int_{x=0} d^2r\;\hat{x}\cdot(\vec{B}\wedge \mathrm{curl}\,\vec{B})\,.$$

where S is the surface area

$$V_3(|\vec{r}_\nu - \vec{r}_\mu|) = \frac{\phi_0^2/\mu_0}{4\pi\lambda_1^2}\;\frac{\exp(-|\vec{r}_\nu - \vec{r}_\mu|/\lambda_1)}{|\vec{r}_\nu - \vec{r}_\mu|} \tag{3.3}$$

is a rotationally symmetric interaction potential between FL elements and

$$V_S(\vec{r}_\nu, \vec{r}_\mu) = \frac{\phi_0^2/\mu_0}{2\lambda_1^2}\int\frac{d^2k_2}{(2\pi)^2}\; e^{-\alpha(x_\nu + x_\mu)}\; e^{i\vec{k}_2\cdot(\vec{r}_\nu - \vec{r}_\mu)} \\ \cdot\Big[\frac{1}{\beta}\Big\{1 - \frac{(k_2 - \beta)}{2\alpha^2(k_2 + \beta)}\Big\} - \frac{1}{\alpha}\Big]\,. \tag{3.4}$$

which is not spherically symmetric but depends on x and $|\vec{r}_2|^2$ separately. This means that V_S is the interaction potential between the x components of the FL elements.

The first two terms in Eq.(3.2) originate only from the FLs field $\vec{B}_\nu$. These terms can be separated into two terms: (i) a bulk term, i.e. half the energy of the interaction of an infinite FL with all FLs and their images expressed as

$$\frac{1}{2}\sum_\nu \oint d\vec{r}_\nu \sum_\mu \oint d\vec{r}_\mu\,[V_3(|\vec{r}_\nu - \vec{r}_\mu|) + V_3(|\vec{r}_\nu - \vec{r}_\mu|)]\,.$$

(ii) A surface term, i.e. twice the interaction between all FLs and their images expressed as

$$-\sum_\nu \int dz \sum_\mu \int dz'\,(y'_\nu y'_\mu + 1)\, V_3(|\vec{r}_\nu - \vec{r}_\mu|)\,.$$

Here, only the y and z components of the FL elements interact with the images.

The third term in the free energy (3.2) originates only from the stray fields $\vec{B}_{S1}$ and $\vec{B}_{S2}$. In this term only the x components of the FL elements interact with the images. Moreover, it decreases exponentially with $(x_\nu + x_\mu)$ and thus the stray field energy originates only from the FLs within a surface layer of thickness $\sim (\lambda_1 + \lambda_2)$.

The last terms in the free energy (3.2) originate only from the London penetrating fields $\vec{B}_{L1}$ and $\vec{B}_{L2}$ which do not depend on the FL positions. For $\lambda_1 < |\vec{r}_\nu| < \lambda_2$, the Gibbs free energy is given by

$$G_2 = \frac{1}{2}\sum_\nu \oint dr_\nu \sum_\mu \Big[\oint dr_\mu\, U_3(|\vec{r}_\nu - \vec{r}_\mu|) - \oint dr_{\mu_1} U_3(|\vec{r}_\nu - \vec{r}_\mu|)\Big] + \\ + \sum_\nu \int dz \sum_\mu \int dz'\, x'_\nu(z)\, x'_\mu(z)\, U_S(\vec{r}_\nu(z), \vec{r}_\mu(z)) - \\ - \frac{1}{\mu_0} S(\lambda_1|\vec{a}|^2 + \lambda_2|\vec{b}|^2), \quad \lambda_1 < |\vec{r}_\nu| < \lambda_2 \tag{3.5}$$

with

$$U_3(|\vec{r}_\nu - \vec{r}_\mu|) = -\frac{\phi_0^2/\mu_0}{4\pi\lambda_2^2}\,\frac{\exp(-|\vec{r}_\nu - \vec{r}_\mu|/\lambda_2)}{|\vec{r}_\nu - \vec{r}_\mu|} . \qquad (3.6)$$

and

$$U_s(\vec{r}_\nu, \vec{r}_\mu) = -\frac{\phi_0^2/\mu_0}{2\lambda_2^2}\int \frac{d^2k_2}{(2\pi)^2}\, e^{-\alpha(x_\nu + x_\mu)}\, e^{i\vec{k}_2\cdot(\vec{r}_\nu - \vec{r}_\mu)} \cdot \left[\frac{1}{\beta}\left\{1 - \frac{(k_2-\beta)}{2\alpha^2(k_2+\beta)}\right\} - \frac{1}{\alpha}\right] . \qquad (3.7)$$

If we assume that both of the two phases (homogeneous HTS) are in the mixed state. in this case the Gibbs free energy takes the form

$$\begin{aligned} G = \frac{1}{2}\sum_\nu \oint dr_\nu \sum_\mu \Big[\oint dr_\mu \{V_3(|\vec{r}_\nu - \vec{r}_\mu|) + U_3(|\vec{r}_\nu - \vec{r}_\mu|)\} - \\ - \oint dr_{\mu 1}\{V_3(|\vec{r}_\nu - \vec{r}_{\mu 1}|) + U_3(|\vec{r}_\nu - \vec{r}_{\mu 1}|)\}\Big] + \\ + \sum_\nu \int dz \sum_\mu \int dz'\, x'_\nu(z)\, x'_\mu(z)\{V_s(\vec{r}_\nu(z), \vec{r}_\mu(z')) + \\ + U_s(\vec{r}_\nu(z), \vec{r}_\mu(z'))\} - \frac{2}{\mu_0} S(\lambda_1|\vec{a}|^2 + \lambda_2|\vec{b}|^2) , \\ \lambda_1, \lambda_2 > |\vec{r}_\nu| \end{aligned} \qquad (3.8)$$

The discontinuity of the FL energy at the interface (i.e. the change in the FL energy at the boundary) is

$$\Delta G = G_1 - G_2$$

with G_1 and G_2 given by Eqs.(3.2) and (3,.5). We have two cases:

i) If ΔG is positive, the FLs will be pinned and there is a potential well at the interface;

ii) If ΔG is negative, there is no pinning of the FLs and there is no potential well at the interface.

4. PINNING FORCE

The FLs are pinned by the boundary between the two phases and outside of this boundary there is no pinning (both of the two phases are considered to be homogeneous HTS). The total pinning force per unit length exerted on the FL element $d\vec{r}_\nu$ is given by

$$d\vec{P}_\nu = \phi_0 \vec{J}(\vec{r}_\nu) \wedge d\vec{r}_\nu \qquad (4.1)$$

$$= d\vec{P}_{1\nu} + d\vec{P}_{2\nu} \qquad (4.2)$$

where $d\vec{P}_{1\nu} = \phi_0 \vec{J}_1(\vec{r}_\nu) \wedge d\vec{r}_\nu$ and $d\vec{P}_{2\nu} = \phi_0 \vec{J}_2(\vec{r}_\nu) \wedge d\vec{r}_\nu$.

Using Eqs.(2.8)-(2.11) one can find

$$\begin{aligned} d\vec{P}_\nu = \frac{c}{4\pi}\phi_0 i \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}_2\cdot\vec{r}_\nu}\Big[\frac{\sigma}{\sigma+1} k_2 (e^{-x_\nu/\lambda_1} - \sigma e^{+x_\nu/\lambda_2}) \\ \cdot\{\hat{x}(\vec{B}_{\nu 2}(\vec{k})\cdot\vec{r}_\nu') - \vec{B}_{\nu 2}(\vec{k})\, x'_\nu\} + B_{\nu x}(\vec{k})\Big\{\Big(\frac{\alpha}{k_2} - 1\Big)e^{-\alpha x_\nu} + \\ + \frac{\alpha}{k_2}\frac{(\frac{\beta}{k_2}-1)}{(\frac{\alpha}{k_2}+1)}\, e^{\beta x_\nu} - \frac{\sigma}{\sigma+1}\frac{\alpha}{k_2(k_2+\alpha)}\Big[\Big(k_2 + \alpha + \frac{2}{\sigma\lambda_1} + \frac{k_2-\beta}{\sigma^2}\Big)e^{-x_\nu/\lambda_1} + \\ + \sigma\Big(k_2 + \alpha - \frac{2}{\lambda_1} - \frac{k_2-\beta}{\sigma^2}\Big)e^{x_\nu/\lambda_2}\Big]\Big\}\{\hat{x}(\vec{k}_2\cdot\vec{r}_\nu') - \vec{k}_2 x'_\nu\} + \\ + e^{ik_x x_\nu}(\vec{k}\wedge\vec{B}_\nu(\vec{k}))\wedge\vec{r}_\nu'\Big]\, dz . \end{aligned} \qquad (4.3)$$

The last term in Eq.(4.2) can be written in the form (using Eq.(2.6))

$$2\Big(\frac{\phi_0}{\mu_0}\Big)\frac{c}{4\pi}\sum_\nu\int\frac{d^3k}{(2\pi)^3}\int\frac{dz}{1+k^2\lambda_1^2}\, e^{ik_x x_\nu} .$$

$$\cdot \begin{pmatrix} -k_x(y'_\nu+1)\sin k_x x_\nu + i x'_\nu(k_x y'_\nu + k_z)\cos k_x x_\nu \\ \{y'_\nu(k_x x'_\nu + k_z) - k_y\}\sin k_x x_\nu - i k_y x'^2_\nu \cos k_x x_\nu \\ \{y'_\nu(-k_z y'_\nu + k_y) + k_x x'_\nu\}\sin k_x x'_\nu - i k_z x'^2_\nu \cos k_x x_\nu \end{pmatrix} dz$$

In the case of the change in the FL energy at the boundary ΔG is positive, the pinning forces, where the FL moves to the right or to the left will be different. This is clear from Eqs.(4.2), (2.8) and (2.9).

5. LIMITING CASES

(i) In the case of straight FLs, the problem will tend to that considered for conventional superconductors by Mkrtchyan et al. [3].

(ii) In the case of $\lambda_2 \to \infty$ (i.e. the phase $x < 0$ will be normal conductors), $\sigma \to 0$ and $\beta \to k_2$. In this case we have the same results obtained by Brandt [4]. In this case $\vec{a}_2 = \vec{B}_{a2}$ - the applied field

$$\vec{B}_{a2} = \vec{a}_2 = -i \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}_2\cdot\vec{r}}\, \frac{2\alpha}{k_2(k_2+\alpha)}\, B_{vx}(\vec{k})\, \vec{k}_2$$

with

$$\begin{aligned} d\vec{P}_\nu = \frac{c}{4\pi}\Big[& \frac{\phi_2}{\lambda_1}\, e^{-x_\nu/\lambda_1} \{ \hat{x}(\vec{B}_{a2}\cdot\vec{r}_\nu') - \vec{B}_{a2}\, x_\nu' \} + \\ & + \phi_0\, i \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}_2\cdot\vec{r}_\nu} \Big[B_{vx}(\vec{k}) \Big(\frac{\alpha}{k_2} - 1\Big) e^{-\alpha x_\nu} \,\cdot \\ & \cdot \{ \hat{x}(\vec{k}_2\cdot\vec{r}_\nu') - \vec{k}_2\, x_\nu' \} + e^{ik_x x_\nu} (\vec{k}\wedge\vec{B}_v(\vec{k}))\wedge\vec{r}_\nu' \Big] dz\,. \end{aligned} \tag{5.1}$$

6. CONCLUSIONS

1. The system of two phases of HTS was considered in order to study the pinning of curved FLs by the interface. This study can help to clarify the extent to which the nature of the pinning forces in HTS can actually be deduced from the experimental data (for two phases of HTS) which have received very limited attention. Furthermore, it may serve to test the pinning experimentally.

2. The FL field is not equal to zero at the interface. For this reason, the stray fields were considered, which must be equal to zero for straight FLs.

3. The transport current can flow along the boundary in a layer of thickness $\sim(\lambda_1 + \lambda_2)$. This current is refracted by the interface, which leads to the interaction of the FLs with the boundary.

4. For positive ΔG, there is a potential well at the interface and the FLs will be pinned by the interface, where there is no pinning outside this interface (we assume that the two phases are homogeneous HTS).

5. When the FL moves to the right or to the left, the pinning forces will be different.

6. In the case of straight FLs, the stray fields must be equal to zero and the problem tends to that which has been considered for two phases conventional superconductors [3].

7. As $\lambda_2 \to \infty$, we obtained the same results for the normal conventional superconductor interface [4].

NOTE ADDED:

After finishing the manuscript, the following paper came to our knowledge: K. Kambara et al. [Physica C 156, 727 (1988)], where clear evidence was presented for the coexistence of two phases of HTS in a ceramic ring of $YBa_2Cu_3O_7$, by measuring electromagnetically transport critical currents.

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