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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

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SOME REMARKS ABOUT QUANTUM GRAVITY *

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1. INTRODUCTION

Gravitation is the oldest of the fundamental interactions known to us. It finds an extraordinarily beautiful and simple formulation in Einstein's General Relativity, which embodies the equivalence principle by requiring the invariance of the theory under general reparametrization of the coordinates $x^\mu \rightarrow x'^\mu = \xi^\mu(x)$. Experiments confirm the predictions of the classical theory in, the macroscopic, large distance, domain.

The recent developments in the description of the fundamental interactions, from gauge theories and spontaneous symmetry breaking to grand unification, have refocused our attention on gravitation as a quantum field theory and on its eventual influence on particle physics.

Quantum effects are expected to become important for very high energies, of the order of the Planck mass $M_{PL} = (G_N)^{-1/2} \approx 10^{19}$ GeV corresponding to distances $L = (4\pi G_N)^{1/2} \approx 10^{-33}$ cm . The dimensional quantity setting the scale of these effects is Newton's constant G_N which, at the same time determines the strength of the macroscopic gravitational phenomena. Unfortunately, such a dimensional character of the gravitational coupling constant makes the resulting theory non renormalizable, namely, higher terms of the conventional perturbation expansion exhibit worse and worse asymptotic behaviour at small distances.

Thus we have to face the problem that a satisfactory understanding of gravity is still lacking at the quantum level, in spite of its certainly unique properties of invariance (1).

Several interesting avenues towards a possible solution have been discussed (some of them in this session) which go from considering gravitation as a spontaneously or dynamically broken theory (2) to the more extreme attempt of a composite model (3). I shall illustrate our proposal (4) which is, in a way, more conservative and tries to exploit fully what a rich theory such as General Relativity, can do for us.

2. CONSIDERATIONS ON DIMENSIONALITY

Let us begin by noticing that the fundamental constant G_N is introduced in the theory in a rather phenomenological way, namely, after all, in order to match the dimensions of the minimal Einstein-Hilbert gravitational action, where $g_{\mu\nu}(x)$ is assumed to be dimensionless. One has

$$A_E = -\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,R \quad . \tag{1}$$

In fact the presence of a dimensional parameter is surprising. Indeed due to its general symmetry properties, the action A_E is in particular invariant under coordinate dilatations, $x'^{\mu} = gx^{\mu}$, and we are accustomed to associate scale invariant field theories with the absence of any dimensional constant. A familiar example is represented by Yang-Mills theories which are invariant under scale transformations in the limit of vanishing masses for all matter fields.

The point presumably lies in the fact that for the particle physicist General Relativity has to be considered as the reparametrization invariant field theory of a massless, spin two particle, the graviton. The geometrical aspect ($g_{\mu\nu}(x)$ as the space-time metric tensor), even if highly suggestive at the classical level, is not essential and $g_{\mu\nu}(x)$ has finally to be treated as a (quantum) field[5]. In particular, a closer look at the general transformation properties of the fields suggest that the scale dimension (we may refer to it as "group theoretical") to be assigned to $g_{\mu\nu}$ is actually - 2 (in units of length). More precisely, one is led to ascribe the following values to the scale dimension of various field quantities:

$$\begin{array}{lcl} g_{\mu\nu},\ g^{\mu\nu},\ \sqrt{-g} & : & \Delta = -2, 2, -4 \\ R_{\mu\nu},\ R = g^{\mu\nu}R_{\mu\nu} & : & \Delta = -2, 0 \\ A_{\mu},\ \phi,\ \psi & : & \Delta = 0 \end{array} \tag{2}$$

where A_{μ}, ϕ, ψ are vector, scalar and spinor fields respectively.

The argument is as follows: given the general coordinate reparametrization

$$x^{\mu} \rightarrow x'^{\mu} = f^{\mu}(x) \simeq x^{\mu} + \varepsilon^{\mu}(x) \tag{A.1}$$

we recall that the corresponding infinitesimal variation of a tensor $t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M}$ is

$$\delta t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M} \equiv \left(t'(x) - t(x)\right)^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M} = -\varepsilon^{\lambda}\partial_{\lambda} t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M} - \tag{A.2}$$

$$- t^{\nu_1\dots\nu_N}_{\lambda\dots\mu_M}\,\partial_{\mu_1}\varepsilon^{\lambda} - \dots + t^{\lambda\dots\nu_N}_{\mu_1\dots\mu_M}\,\partial_{\lambda}\varepsilon^{\nu_1} \dots + t^{\nu_1\dots\lambda}_{\mu_1\dots\mu_M}\,\partial_{\lambda}\varepsilon^{\nu_N} .$$

Let us now consider the special case of a scale transformation

$$\varepsilon^{\mu}(x) = -\varepsilon x^{\mu} \quad . \tag{A.3}$$

From eq. (A.2) one obtains immediately

$$\delta t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M} = \varepsilon(x\cdot\partial - \Delta)\, t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M} \quad , \tag{A.4}$$

where $\Delta = N - M$ is the difference between the number of controvariant and covariant components respectively. Given the form of eq. (A.4) it seems natural to assert that the tensor $t^{\nu_1\dots\nu_N}_{\mu_1\dots\mu_M}$ has dimensionality Δ. Consequently we find the values of dimensionality listed in eq. (2).

In order to avoid any confusion, it is important to mention that the above dilatational transformations are different from the so-called Weyl local scale transformations

$$g_{\mu\nu}(x) \rightarrow g'_{\mu\nu}(x) = \Omega^{-2}(x)\, g_{\mu\nu}(x) \tag{A.5}$$

(in infinitesimal form

$$\delta g_{\mu\nu}(x) \simeq -2\omega(x)\, g_{\mu\nu}(x)) \quad .$$

The implementation of this additional symmetry would require a more complicated structure of the action. The connection between the two types of transformations and their relevance for the flat limit is discussed for instance in Ref. (6).

One is immediately convinced that, as a consequence, the Einstein Hilbert action can be written in a full scale invariant form where no dimensional parameters appear and no additional fields need to be introduced(*). We simply have:

$$A_E = -\frac{1}{4}\int d^4x \sqrt{-g}\, R \, . \tag{3}$$

Naturally, on pure invariance grounds, additional pieces are possible such as a cosmological term $\lambda\sqrt{-g}$, λ dimensionless, or terms proportional to $\sqrt{-g}\,R^2, \sqrt{-g}\,R_{\mu\nu}R^{\mu\nu}$ which can be discarded on the basis of a minimality requirement (to avoid four derivatives and ghost poles in the propagator).

Notice the essential role of the non polynomial field quantity $\sqrt{-g}$ in the matching of dimensions.

3. THE VACUUM

All this may look a little tricky and the natural question at this point is "where has Newton's constant gone?" Our proposal is that Newton's constant finds its natural place, not as a coupling constant in the action but in the boundary condition which specifies the behaviour of $g_{\mu\nu}(x)$ at large distances. More precisely when requiring that $g_{\mu\nu}(x)$ asymptotically behaves like the flat solution(**), its form is taken to be

$$g_{\mu\nu}(x)\Big|_{flat} = \frac{1}{L^2}\eta_{\mu\nu} \, , \qquad L^2 = 4\pi G_N \, . \tag{4}$$

(*) Our approach is therefore different from the one advocated by several authors (see talk given by A. Zee at this Conf.), which assumes the existence of a new scalar field whose vacuum expectation value gives rise to Newton's constant.

(**) of the equation of motion $R_{\mu\nu} = 0$.

The constant reproduces at the elementary level the dimension -2 of the field.

The essence of this point of view is that the Newton constant with its dimensionality does not represent a general feature of the gravitational action but rather characterizes the particular and fundamental class of solutions to be used in the description of phenomena at large distances. It is thus quite clear that such a formulation is completely equivalent to the conventional one as far as the "Newtonian" results of general relativity are concerned, however the underlying framework is much more general and offers the possibility of describing "non Newtonian" phenomena, not weighted by G_N.

This argument finds a meaningful and appropriate reformulation in the quantum language. The heart of the matter is to assume that Newton's constant appears in the theory via the vacuum expectation value of $g_{\mu\nu}(x)$ namely

$$\langle 0|\, g_{\mu\nu}(x)\, |0\rangle = \frac{1}{L^2}\eta_{\mu\nu} \tag{5}$$

which is the quantum version of eq. (4).

The situation is reminiscent of the case of the linear σ-model[7] (even if analogies should not be pushed too far!), where the starting Lagrangian has chiral isospin $SU(2)\times SU(2) \sim SO(4)$ as (internal) symmetry group. The vacuum has however a lower symmetry i.e. $SU(2) \sim SO(3)$ and this is clearly exhibited by the existence of a non vanishing vacuum expectation value of the bosonic field ϕ_α $(\alpha = 1,2,3,4)$ namely

$$\langle 0|\, \phi_\alpha(x)\, |0\rangle = f_\pi\, \delta_{\alpha 4} \tag{6}$$

The dimensional constant f_π is a measure of the spontaneous breaking of the symmetry $SO(4)$. As is well known, f_π determines the low energy behaviour of the pion amplitudes and acts in this domain as a universal coupling of pions to any hadronic system while the role of the usual Lagrangian coupling constants is less important.

In a similar way, one can say that while the gravitational action is general invariant the vacuum is not; the flat vacuum solution (5) in particular breaks invariance under dilatations (and thus general invariance) leaving Poincaré as symmetry subgroup. Thus the dimensional constant which rules the large distance behaviour of the transition amplitudes for emission and absorption of gravitons does not appear in the lagrangian. Lagrangian constants, like a possible cosmological term, do not seem to play any important role in classical gravitation.

These considerations also maintain their validity when matter is present, and once more only dimensionless constants have to appear in the total Lagrangian.

It is instructive to examine in a little more detail the case where a matter field is present. This will help us to understand the apparent clash between the above value of zero dimensionality for scalar and spinor fields and the canonical ones (-1 and - 3/2 respectively).

Let us consider the simple example of a scalar field: the action is

$$A_M = \int d^4x \sqrt{-g}\left\{ \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + P(\phi) \right\} \tag{B.1}$$

and $P(\phi)$ is an arbitrary polynomial

$$P(\phi) = \lambda_2\phi^2 + \lambda_3\phi^3 + \lambda_4\phi^4 + \dots \tag{B.2}$$

where, given the dimension zero of ϕ, the λ_i's are pure numbers. The fact that all these terms are allowed may seem surprising. This is clarified if one goes to the flat limit which corresponds to $g_{\mu\nu} = 1/L^2\,\eta_{\mu\nu}$ and

$$A_M \to \int d^4x \left\{ \frac{1}{L^2}\,\frac{1}{2}\eta_{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{L^4}P(\phi) \right\} . \tag{B.3}$$

We thus see that in order to recover in the flat limit the familiar kinetic term, independent of L, a new field has to be used

$$\varphi(x) = \frac{1}{L}\phi(x) . \tag{B.4}$$

A more refined discussion shows that the appropriate field to be used for flat space theories is actually

$$\varphi(x) = (-g)^{1/8}\phi(x) \tag{B.5}$$

a density of dimensionality - 1.

What about the polynomial part and its flat limit? One has

$$\frac{1}{L^4}P(L\phi) = \frac{\lambda_2}{L^2}\varphi^2 + \frac{\lambda_3}{L}\varphi^3 + \lambda_4\varphi^4 + \dots \tag{B.6}$$

Thus a mass term $(\lambda_2/L^2)^{1/2}$ appears, of the order of the Planck mass, together with couplings of the super-renormalizable, renormalizable, and un-renormalizable types.

The connection with the usual (flat) theory of conformal invariance is subtle and the main motivation of the wise choice (B.5) is to achieve it. In fact, it is possible to show that the energy-momentum tensor for a flat space takes on the form

$$\theta_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{4}\eta_{\mu\nu}(\partial\varphi)^2 + \frac{1}{L^4}\eta_{\mu\nu}\left[P - \frac{1}{4}\varphi\frac{dP}{d\varphi}\right] \tag{B.7}$$

so that

$$\theta_\mu^{\ \mu} = \frac{1}{L^4}\left(4P - \varphi\frac{dP}{d\varphi}\right) \tag{B.8}$$

Conformal invariance requires the trace $\theta_\mu^{\ \mu}$ to vanish: then only the term is allowed, as we are used to.

As a conclusion all masses are measured in units of the Planck mass and are therefore related to a spontaneous breaking of dilatation. It is amusing to notice that in this spirit the universal gravitation law can be written in a form similar to the Coulomb one, i.e. in terms of dimensionless gravitational "charges" $\mu_i = m_i L = m_i\sqrt{4\pi G_N}$:

$$U(r) = -G_N\frac{m_1 m_2}{r} = -\frac{\mu_1\mu_2}{4\pi r} . \tag{B.9}$$

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4. RENORMALIZABILITY AND SMALL DISTANCE BEHAVIOUR

The fact that the fundamental Lagrangian is naturally scale invariant is expected to have some important consequences. First of all, since G_N characterizes a solution rather than the full theory, different classical solutions can exist which obey non Newtonian boundary conditions and which may be used as background to describe phenomena occurring in different space-time domains.

It is well known in fact that classical solutions exist for gravitation with a cosmological term and/or interacting with matter fields (gauge fields, non linear σ-model etc.). These solutions are non-Newtonian in the way explained above and bear a dependence on dimensional constants which are always introduced through the boundary conditions. Such constants can be considered as characterizing different vacua, whose relevance to a study of the hadronic structure is an open matter and we shall not discuss it herein(*).

Secondly, but more important, one can expect that some general quantum features of the Green functions, like the small distance behaviour or the commutation relations, should be almost independent of L and substantially be fixed by the invariance properties of the underlying action. In order to discuss this point it is useful to recall how the presence of Newton's constant leads, in the conventional formulation, to the non-renormalizability of quantum gravity. Applying standard perturbative techniques to gravitation is not immediate due to the non-polynomial character of the Lagrangian, which contains the inverse operator $g^{\mu\nu}$ and complicated animals like $\sqrt{-\det g_{\mu\nu}}$ etc. The problem is usually tackled by separating the field $g_{\mu\nu}$ in a Newtonian background plus a quantum part:

$$g_{\mu\nu}(x) = \frac{1}{L^2}\left(\eta_{\mu\nu} + L\varphi_{\mu\nu}(x)\right) . \tag{7}$$

$\varphi_{\mu\nu}$ is an operator of dimension -1 and the subsequent procedure consists of an expansion in $L\varphi_{\mu\nu}$. It thus follows that the interaction term is actually a power series in L: being L a dimensional coupling constant, one faces all the unpleasant peculiarities of a non-renormalizable theory.

In our opinion, this is an unavoidable consequence of using, at small distances, the expansion in L which is only suitable for a description at large distance. Taking into account only a finite number of terms in this expansion clearly violates the general invariance properties of the theory (only Poincaré invariance is respected); these indeed require a non-polynomial Lagrangian and such a character is preserved only after summing the whole series. Therefore it is not surprising that the use of the above expansion beyond its limits of validity does not reproduce the correct small distance behaviour.

This discussion also provides a hint about the relative importance of the effects determined by the vacuum on one side, and by the general invariance of the theory on the other. The main conclusion is that the vacuum essentially represents an infrared phenomenon: it depends on L (or on other constants according to the physical situation) and is an extra information independently added to the Lagrangian. On the contrary, the ultraviolet behaviour will substantially be inferred from the general properties of the underlying Lagrangian only and, consequently, there is no dependence (or a soft one) of these results on L. (The additional problem of smoothly joining these two aspects is completely open and an understanding of it has still to come, as is the case in Q.C.D.).

It is fruitful to illustrate these considerations with some explicit examples. The crux of the matter is represented by the use of inverse operators and we shall firstly consider the case of a free scalar field, of zero mass and dimension -1. We expand it around a (constant) classical background μ namely

$$h(x) = \mu + h'(x) \quad , \qquad \langle h'(x)\rangle_0 = 0 \tag{8}$$

and its Green function is of the form (as experts know, it is useful to work in an Euclidean metric)

$$\langle h(x)h(y)\rangle_0 = \mu^2 + G(x-y) = \mu^2 + \frac{1}{4\pi^2(x-y)^2} . \tag{9}$$

This simple result already suggests that at large distances the Green function is given by the properties of the vacuum (the constant part) while, at small distance, the leading term is $[(x-y)^2]^{-1}$, in agreement with an elementary dimensional argument.

(*) Excellent reviews exist of classical solutions and we need not repeat them[8]. It is however interesting that the space-time dependence of some solutions allows one to reproduce the elementary dimension -2 of $g_{\mu\nu}$ without introducing G_N.

For instance, in the case of gravitation coupled to a gauge field and with a cosmological term λ we can mention, as the simplest example, the so-called meron solution which reads (in an Euclidean metric)

$$g^M_{\mu\nu}(x) = \delta_{\mu\nu}\, 2/\lambda^2 \quad 1/x^2$$

with $\lambda^2 \approx e^2$ being the gauge charge.

The interesting question arises when one wants to evaluate the two-point function of the inverse operator $h^{-1}(x)$.

The simplest way to obtain it is through an expansion in h'/μ :

$$I(x-y) \equiv \langle h^{-1}(x)\, h^{-1}(y)\rangle_0 = \left\langle \frac{1}{\mu+h'(x)}\;\frac{1}{\mu+h'(y)}\right\rangle_0 = \tag{10}$$

$$= \frac{1}{\mu^2}\left\langle \sum_n \left(-\frac{h'(x)}{\mu}\right)^n \sum_m \left(-\frac{h'(y)}{\mu}\right)^m \right\rangle_0 = \frac{1}{\mu^2}\sum_n n!\left[\frac{G(x-y)}{\mu^2}\right]^n$$

Using the integral representation

$$n! = \int_0^{\infty} \alpha^n e^{-\alpha}\, d\alpha \tag{11}$$

we easily reach the already known result[9]

$$I(x-y) = \int_0^{\infty} \frac{e^{-\alpha}\, d\alpha}{\mu^2-\alpha G(x-y)} \tag{12}$$

The correct definition of $I(x-y)$ actually requires a further prescription for the behaviour at the pole (it turns out that the principal value is the right recipe); however, since we are interested in the small distance behaviour this point, which would affect the spectrum properties, is not of immediate interest to us.

In particular as $(x-y)\to 0$ we obtain from eq. (12) that

$$I(x-y) \sim (x-y)^2 \ln[\mu^2(x-y)^2] \;. \tag{13}$$

Therefore, apart from the logarithmic term which contains a residual dependence on the background, the ultraviolet behaviour is the one we should have expected from a simple dimensional count. Notice however that this result required summing the complete series in $1/\mu^2$: taking into account only a finite number of terms would, on the contrary, lead to the completely wrong indication of a theory dramatically divergent at each order of the

perturbation expansion. Similar results stand if more complicated cases are considered, for instance

$$\langle h^{-n}(x)\, h^{-n}(y)\rangle_0 \underset{x-y\to 0}{\sim} [(x-y)^2]^n \ln[\mu^2(x-y)^2] \tag{14}$$

(The logarithmic factor always appears in the first power.)

In the case of gravity the problem is, of course, far more complicated and we have no complete answer concerning full renormalizability. However the analysis of some simple examples shows that dimensional arguments still keep their validity.

Let us introduce for convenience the vierbein fields $V_\mu^a(x)$, $\left(g_{\mu\nu} = \sum_{a=1}^{4} V_\mu^a(x) V_\nu^a(x)\right)$ of dimension -1; then the use of known integral representations for a determinant lead to results which confirm the above point of view, for instance

$$\langle (\det V_\mu^a(x))^{-1} (\det V_\nu^b(y))^{-1}\rangle_0 \sim \underset{x-y\to 0}{\sim} [(x-y)^2]^4 \ln[\mu^2(x-y)^2] \tag{15}$$

and similar.

Notice that results like (13),(14),(15) are quite general in the sense that they depend very little upon dividing the fields into a classical background plus a quantum part. On the other hand, for practical purposes it seems almost unavoidable to rely on a perturbative-like expansion of some kind. In doing so, much care has to be taken, as the previous example shows, before inferring any conclusion on the full theory from the behaviour of a finite number of terms of the series; for this purpose the choice of the initial separation of the field plays a crucial role.

Indeed the usual representation (the analogue of eq. (7))

$$V_\mu^a(x) = \frac{1}{L}\delta_{\mu a} + V_\mu'^{a}(x) \tag{16}$$

is a good starting point only in the infrared domain while at small distances the expansion in $\mathcal{L}_1$ leads to the well-known difficulties.

Considering this, this fact is not that surprising since the separation (16) is Poincaré invariant but not scale invariant (the two pieces transform differently) and cannot, therefore, verify arguments based on dilatations. In order to investigate the ultraviolet regime a conformal and scale invariant decomposition is more suitable. A possibility is to write[10]

$$V_\mu^a(x) = \delta_{\mu a} h(x) + \bar{V}_\mu^a(x) \quad , \tag{17}$$

where $h(x)$, $\bar{V}_\mu^a(x)$ are both operators of dimension -1, and to expand in $\bar{V}/h$ an object of dimension zero. Its two-point function behaves logarithmically and does not substantially influence the theory at small distances:

$$\left\langle \bar{V}_\mu^a(x)/h(x) \quad \bar{V}_\nu^b(y)/h(y) \right\rangle_0 \underset{x-y\to 0}{\sim} \ln\left[\mu^2(x-y)^2\right] \quad . \tag{18}$$

More precisely, the interaction Lagrangian turns out to be of the form

$$\mathcal{L}_1 = \partial_{\mu_1} V_{\nu_1}^{a_1} \partial_{\mu_2} V_{\nu_2}^{a_2} F^{\mu_1 \nu_1 a_1}_{\mu_2 \nu_2 a_2} (\bar{V}/h) \quad . \tag{19}$$

The function $F_{\dots}(\bar{V}/h)$ can be expressed as a series in $\bar{V}/h$ so that $\mathcal{L}_1$ has dimension -4 in any order and should not behave differently from a renormalizable model, apart from first order logarithmic terms like in (18).

The model is highly heuristic, with some internal difficulties ($h(x)$ is a ghost, just to mention one), however it has the good quality of concretely illustrating a possible hint towards the solution of the renormalizability problem of quantum gravity.

5. CONCLUSIONS

Let us summarize the main points of our approach. These are

a) to ascribe the correct dimensionality to the field $g_{\mu\nu}$ (or $V_\mu^a(x)$): this looks quite natural.

b) to interpret Newton's constant as a vacuum effect: this requires distinguishing between boundary conditions and fundamental laws, as we are getting accustomed to in particle physics.

c) to treat inverse operators, a problem which has already attracted some attention for effective chiral Lagrangians(*).

As a follow up of all this we expect that at small distances the Green functions of quantum gravity are substantially of a power form(**) with the exponent determined by purely dimensional arguments: gravitation should not behave differently from a renormalizable theory. Perhaps we may be able to paraphrase Coleman "The divergence structure of a field theory respects the symmetry of the Lagrangian even if the vacuum does not"[11].

The statement by Coleman of course refers to renormalizable polynomial theories with internal symmetries but it is an extremely interesting problem to ascertain whether similar considerations apply to the non-polynomial case with space-time symmetries.

In this spirit we should like to conclude by saying that the problem of renormalizing quantum gravity is, in our opinion, still open and that the same point can be made about the importance of quantum gravitational effects at very high energies.

(*) It is quite interesting that all these features arise naturally in the model of $g_{\mu\nu}$ as a composite field recently proposed by Amati and Veneziano[3].

(**) Apart from logarithmic terms.

References

1. For a recent review see "Recent developments in gravitation" Cargése Summer Institute 1978 Plenum Press N.Y. (1979).

2. A.D. Sakharov, Dokl. Akad. Nauk, SSR 177 (1967) 70;(Soviet Physics Dokl. 12 (1968) 1040);
Ya.B. Zel'dovich, Zh.ETF Pis. Red. 6 (1967) 922; (JETP Lett. 6 (1967) 345);
O. Klein, Phys. Scr. 9 (1974) 69;
K. Akama, Y. Chikashige, T. Matsuki and H. Terazawa, Progr. Theor. Phys. 60 (1978) 868;
K. Akama, Progr. Theor. Phys. 60 (1978) 1900;
S.L. Adler, Phys. Rev. Lett. 44 (1980), Phys. Lett. 95B (1980) 241;
B. Hasslacher and E. Mottola, Phys. Lett. 95B (1980) 237;
A. Zee, Phys. Rev. D23 (1981) 858.

3. D. Amati and G. Veneziano: "Metric from matter" CERN preprint, Ref. TH 3126(1981)

4. V. de Alfaro, S. Fubini and G. Furlan: Il Nuovo Cimento A50, 523 (1979) and Il Nuovo Cimento B57, 227 (1980).

5. See for instance, S. Weinberg : "Gravitation and Cosmology" J. Wiley, New York, (1972);
A. Salam: "Impact of quantum gravity in Particle Physics" in "Quantum Gravity, an Oxford Symposium" eds. C.Isham, R. Penrose, D. Sciama, Oxford Univ. Press (1975).

6. J. Wess: "Conformal invariance and the energy-momentum tensor" SPringer Tracts in Modern Physics Vol. 60 (1971).

7. See for instance V. de Alfaro, S. Fubini, G. Furlan and C. Rossetti: "Currents in Hadron Physics" North Holland Publ. Amsterdam (1973).

8. Several aspects of classical solutions are reviewed in V. de Alfaro, S. Fubini and G. Furlan, Acta Physica Austriaca Supp. XXII 51, 1980.

9. A quite complete presentation canbe found in "Non-polynomial Lagrangians, renormalization, gravity", Tracts in Mathematics and Natural Sciences Vol. 1, 1971, A.Salam ed.

10. V. de Alfaro, S. Fubini and G. Furlan: Phys. Lett. B97, 67 (1980).

11. S. Coleman in "Laws of Hadronic Matter" A. Zichichi ed., Academic Press,N.Y. (1975).

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